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ALEXANDER KOEBERLEIN

Elementary Geometry for COLLEGE STUDENTS 7e

7E

Elementary Geometry

for College Students

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***Elementary Geometry for College Students*, Seventh Edition**
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This edition is dedicated to our spouses, children, and grandchildren.
Dan Alexander and Geralyn Koeberlein

LETTER FROM THE AUTHORS

The primary aim of this textbook has always been to provide college students with a thorough knowledge of the subject matter of geometry. We thank all those instructors who agreed with our approach to the development of geometry. Further, we applaud all those students who have persevered (with our textbook as the vehicle) and achieved a substantial knowledge of geometry; in fact, it has been said that there is no royal road (easy way) to geometry.

When we began formulating the first edition of this textbook, we sought to achieve and maintain many qualities throughout future editions. One of the foremost goals that we sought was that of creating a logical and orderly development of geometry. Although we have managed goal one successfully, we also believe that we have improved our presentation with time: formal logical and visual reasoning have been apparent through previous editions. In effect, a student of geometry must "see" to believe.

A second goal was to find the perfect choice of words essential in guiding the student down the path to geometry. For the student to read and fully understand the notions found in this development, it was generally necessary to illustrate these terms visually.

A third goal of our textbook has been to provide examples and practice (exercise sets) that would reinforce the embedded concepts. While the textbook provides abundant practice, further exercises can be found in the *Student Study Guide*. Many examples, features, and exercises demonstrate applications of geometry in the real world. In exercise sets, problems are ordered from basic to intermediate and then challenging.

In this seventh edition, we have at times clarified a definition by a change in wording or by an example/illustration. In some cases, we have found a clearer (more direct path) to one of the many principles of geometry. Through modifying an explanation, we facilitated the reasoning process used to understand and formulate proofs, solve problems, and handle real-world applications. Such tools enable students to use (1) intuition to guess results, (2) induction to test these outcomes, and (3) deduction to prove the relationships found in geometry. To reinforce the student's knowledge of geometry, we have developed a textbook that may preview a concept or recall/review (such as review exercises at a chapter's end). Within a section, topics are often accompanied by hints, recalls, and guides designed to provide insights into the steps in an example, the plan for a proof, or the solution of an exercise.

For students to reach the lofty goals of this textbook, it is essential that they enter the study with open minds and accept the challenge. Successful students will develop a knowledge of geometry and the associated skills to allow them to advance their mathematical and related disciplines.

Daniel C. Alexander and Geralyn M. Koeberlein

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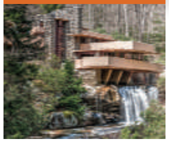

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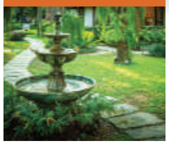

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Preface

The authors' primary goal is that of helping and encouraging students of the seventh edition of *Elementary Geometry for College Students* to learn and use the terminology of geometry, explore the principles of geometry, develop strong reasoning skills, gain new skills in problem-solving, and find solutions for real-world applications. We hope not only to introduce students to geometry, but to have them respect and enjoy the subject matter.

The authors have developed the textbook in a logical order that is complete with features that are intuitive and motivating. Perhaps the most significant feature of this textbook is that it provides seemingly countless visual illustrations; of course, the picture may very well be worth a thousand words.

We, the authors, truly believe that the student who successfully completes the geometry coursework with this textbook will be able to, at a future date, recognize and recall the relationships of geometry, understand and justify the connections between certain properties, and extend and apply these principles.

Justification of a principle will be done in a formal two-column form, a formal paragraph form, or in the less formal "picture proof" format.

We are well aware that the completion of a proof is quite the challenge for most students. With this in mind, we hope to illustrate the power of the proof so that the student gains an appreciation for this power. To begin, we ask that the student simply follow the illustrated proof, reading it step-by-step. Reading the proof in reverse order enables the student to answer the question, "Where are we coming from?" Eventually, the student provides missing pieces (statements and reasons) of the proof. In time, the student realizes the need to order and justify the steps of the proof. Of course, the ultimate goal is that the student writes the complete proof. The authors believe the logic found in development of geometry extends to other disciplines; for instance, consider writing a good paragraph in literature or a powerful subroutine in computer science. One should recognize that the principles, the thoughts, and the reasoning that lead to a proof are also used in the solution of any problem or application.

OUTCOMES FOR THE STUDENT

- Mastery of essential geometric concepts, for further intellectual and vocational endeavors
- Preparation of the student to transfer and continue the study of mathematics and related disciplines
- Understanding of the need for step-by-step reasoning for success in geometry and other disciplines
- Development of their interest in geometry through activities, features, and problem-solving

FEATURES OF THE SEVENTH EDITION

- Inclusion of Chapter P, which provides an expanded introduction of set concepts and formal reasoning
- Increased attention to continuity and discontinuity of geometric figures
- Expanded coverage of point paths characterized as straight, curved, circular, or scattered
- Revision of existing exercise sets as well as the inclusion of over 150 new exercises
- Expanded coverage of select topics, such as the discussion of overlapping congruent triangles
- Greater attention to the congruence of quadrilaterals.
- Revision of select paragraphs and the addition of examples in order to clarify concepts and approaches

TRUSTED FEATURES

Full-color format aids in the development of concepts, solutions, and investigations throughout all figures and graphs. The authors have ensured that color in all figures is both accurate and instructionally meaningful.
Reminders found in the text margins provide a convenient recall mechanism.
Discover activities emphasize the importance of induction in the development of geometry.
Geometry in Nature and **Geometry in the Real World** demonstrate geometry found in everyday life.
Overviews in chapter-ending material organize important properties and other information from the chapter.
An Index of Applications calls attention to the practical applications of geometry.
A Glossary of Terms at the end of the textbook provides a quick reference of geometry terms.
Chapter-opening photographs highlight subject matter for each chapter.
Warnings alert students to common pitfalls.
Chapter Summaries review the chapter, preview the chapter to follow, and provide a list of important concepts found in the current chapter.
Perspective on History provides students with biographical sketches and background leading to geometric discoveries.
Perspective on Applications explores classical applications and proofs.
Chapter Reviews provide numerous practice problems to help solidify student understanding of chapter concepts.
Chapter Tests provide students the opportunity to prepare for exams.
Formula pages at the front of the book list important formulas with relevant art to illustrate.
Reference pages at the back of the book summarize the important abbreviations and symbols used in the textbook.

STUDENT RESOURCES

- **The Student Study Guide with Solutions Manual** (978-0-357-02212-2) provides step by step solutions to select odd-numbered exercises from the text. Select solutions for additional exercise sets are provided within the study guide. Complete solutions are available on the instructor's website.
- WEBASSIGN **www.webassign.com**
  Prepare for class with confidence using WebAssign from Cengage *Elementary Geometry for College Students*, *Seventh Edition*. This online learning platform fuels practice, so you truly absorb what you learn—and are better prepared come test time. Videos and tutorials walk you through concepts and deliver instant feedback and grading, so you always know where you stand in class. Focus your study time and get extra practice where you need it most. Study smarter with WebAssign!
  Ask your instructor today how you can get access to WebAssign, or learn about self-study options at www.webassign.com.

INSTRUCTOR RESOURCES

- WEBASSIGN **www.webassign.com/cengage**
  WebAssign from Cengage *Elementary Geometry for College Students*, *Seventh Edition*, is a fully customizable online solution for STEM disciplines that empowers you to help your students learn, not just do homework. Insightful tools save you time and highlight exactly where your students are struggling. Decide when and what type of help students can access while working on assignments—and incentivize independent work so help features aren't abused. Meanwhile, your

students get an engaging experience, instant feedback, and better outcomes. A total win-win!

To try a sample assignment, learn about **LMS** (Learning Management System) integration, or connect with our digital course support, visit www.webassign.com/cengage.

- **Complete Solutions Manual** provide solutions to all the exercises in the book, alternatives for order of presentation of the topics included, and suggestions for teaching each topic.
- **Cognero** (Included on the Instructor Companion Website) Cengage Learning Testing Powered by Cognero is a flexible, online system that allows you to author, edit, and manage test bank content from multiple Cengage Learning solutions; create multiple test versions in an instant; and deliver test from your LMS, your classroom, or wherever you want.

INSTRUCTOR WEBSITE

Everything you need for your course in one place! This collection of book-specific lecture and class tools is available online at www.cengage.com/login. Here you can access and download PowerPoint presentations, images, and more.

ACKNOWLEDGMENTS

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Foreword

For the seventh edition of *Elementary Geometry for College Students*, the topics that comprise a minimal course include most of Chapters 1–6 and Chapter 8. For a more comprehensive course, the inclusion of Chapters 1–8 is recommended. Chapter P is recommended for students who have not had an introduction to sets and methods of reasoning. Considering time constraints in the course content, these topics could be treated as optional:

- Section 2.6 Symmetry and Transformations
- Section 3.4 Basic Constructions Justified
- Section 3.5 Inequalities in a Triangle
- Section 5.6 Segments Divided Proportionally
- Section 6.4 Some Constructions and Inequalities for the Circle
- Section 7.1 Locus of Points
- Section 7.2 Concurrence of Lines
- Section 7.3 More About Regular Polygons
- Section 8.5 More Area Relationships in the Circle
- Section 10.6 The Three-Dimensional Coordinate System

To determine whether the study of Chapter P should be excluded (included), an instructor (or student) should consider the simplicity (difficulty) of the chapter test found at the end of Chapter P. Considering that this textbook may be used for a course of 3, 4, or 5 credit hours, the following diagram indicates the possible orders in which the chapters of the textbook can be used:

$$1 \rightarrow 2 \rightarrow 3 \rightarrow 4 \rightarrow 5 \rightarrow 6 \rightarrow 7 \text{(or 8)} \rightarrow 8 \rightarrow \text{(and/or 9 and/or 10 and/or 11)}$$

For students who need further review or explanation of related algebraic topics, consider those found in Appendix A:

A.1 Algebraic Expressions

A.2 Formulas and Equations

A.3 Inequalities

A.4 Factoring and Quadratic Equations

A.5 The Quadratic Formula and Square Root Properties

Sections A.4 and A.5 include these methods of solving quadratic equations: the factoring method, the square roots method, and the Quadratic Formula.

Daniel C. Alexander and Geralyn M. Koeberlein

Index of **Applications**

7E

Elementary Geometry

for College Students

Preliminary Concepts

P

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CHAPTER OUTLINE

Foundations! The building blocks for the study of geometry can be found in the early chapters. A firm foundation is essential for the building of knowledge in geometry or any other field. However, you may already be familiar with the concepts in Chapter P; if so, begin your study with Chapter 1. In Section P.1, we introduce the notion of sets (collections of objects). Sets of points form all geometric figures. For instance, lines, angles, triangles, and circles are sets of points. Regarding the geometric figures described above, we will study them, reason about them, and thus develop their properties. In Section P.2, we introduce three types of reasoning that are fundamental to the development of geometry. The process of developing valid properties about geometric figures occurs in this order:

1. We have an idea (intuitive reasoning).
2. We repeatedly test the idea (inductive reasoning) to be sure that it is credible.
3. We verify the idea/relationship as a necessary conclusion by piecing together, in order, claims that we have already established.

Many fundamental concepts of geometry are introduced informally in Section P.3.

P.1 Sets and Geometry

KEY CONCEPTS

Set
Element
Finite and Infinite Sets
Subset
Point Paths (Straight, Curved, Circular, Scattered)
Between
Continuous/Discontinuous
Line Segment, Line, Ray
Intersection and Union of Sets
Empty Set
Angle
Disjoint Sets
Venn Diagrams
Equivalent Sets
Universe
Complement of Set

SETS AND SETS OF NUMBERS

A **set** is any collection of objects, such as a collection of numbers or a collection of points. The objects in a set are known as the **elements** of the set. Some sets are collections of numbers such as the set of natural numbers or counting numbers $N = \{1,2,3,4, \ldots\}$, which is read "the set containing 1, 2, 3, 4, and so on." Another well-known set is $R = \{\text{real numbers}\}$, the set of numbers on the number line; the real numbers include positives and negatives, whole numbers, common fractions, and irrational numbers such as $\sqrt{2}$ and π. Decimal forms for common fractions terminate or repeat, while those for irrational numbers do neither. In the following example, a calculator is used to find decimal forms.

EXAMPLE 1

Find the decimal forms for these numbers:

a) $\frac{3}{4}$ **b)** $\frac{3}{11}$ **c)** $\sqrt{2}$ **d)** π

SOLUTION

a) $\frac{3}{4} = 0.75$ **b)** $\frac{3}{11} = 0.2727\ldots$

c) $\sqrt{2} = 1.4142135\ldots$ **d)** $\pi = 3.1415926\ldots$

Some sets are **finite**; such sets have a countable number of elements. At times, we use the symbol $N\{A\}$ to represent the number N of elements in a finite set A; for instance, $N\{1,2,3\} = 3$. Many sets are **infinite**; these sets have an uncountable number of elements (too many elements to count).

EXAMPLE 2

Are these sets finite or infinite?

a) $V = \{\text{vowels}\} = \{a,e,i,o,u\}$ **b)** $N = \{\text{natural numbers}\}$

SOLUTION

a) V is finite; V has 5 elements, expressed by the statement $N\{V\} = 5$.
b) N is infinite; there is no largest counting number.

The Student Study Guide (SSG) will provide additional practice.

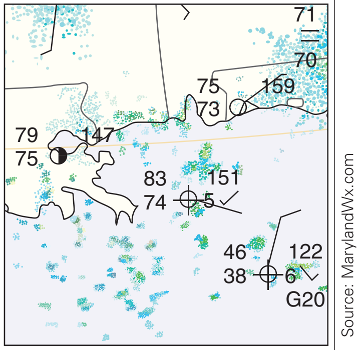
SSG **EXS. 1, 2**

When every element in one set belongs to another set, the first set is called a **subset** of the second set. For instance, if A is the set $A = \{1,2,3\}$, then A is a subset of N, the set of natural numbers (counting numbers); in symbols, $A \subseteq N$.

NOTE: The open part of the subset symbol is toward the larger set.

SETS OF POINTS

Geometry in Nature



The blue "dots" represent showers—evidently of a "scattered" nature.

In geometry, a **point** has location but does not have size. A point is shown as a dot and is generally named by an uppercase letter such as A. In Figure P.1, we see points A, B, and C.

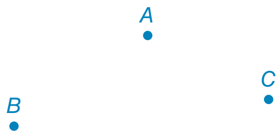


Figure P.1

When many points appear together, a pattern or **path** is often formed. Some of those paths are characterized in Figure P.2. In Figure P.2(a), (b), and (c), you may think of the path in much the same way that you would "stepping stones"; however Figure P.2(d) does *not* create a path.

Figure P.2

When we consider three of the points on a straight path (shown on a line), we generally describe one point as being **between** the remaining two points; that does not necessarily mean halfway between the other points.

Figure P.3

In Figure P.3, point S lies between points R and T.

A line is formed when an infinite number of points follow a straight path. One characteristic of the line is that between any two points, there is always another point. The infinite set of points that form the line is so dense that the line is said to be **continuous**. Thus, a line can be drawn without lifting the pencil from the paper.

Figure P.4

In Figure P.4, line BC (in symbols, $\overleftrightarrow{BC}$) is named by two points B and C on the line.

Some subsets of the line are also shown in Figure P.5.

Name in words	Symbols	Endpoint(s)
Line segment *BC*	$\overline{BC}$ or $\overline{CB}$	*B* and *C*
Ray *BC*	$\overrightarrow{BC}$	*B*
Ray *CB*	$\overrightarrow{CB}$	*C*

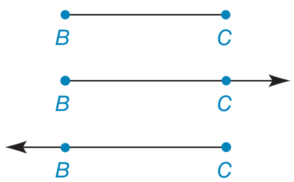


Figure P.5

Later, we will use a ruler to measure (find the length of) a **line segment**; however, a **line** or **ray** does not have a length.

In Figure P.2, the paths created by the points display a quality known as **discontinuous**. We now recall those paths in Figure P.6(a), (b), and (c), but display them with the quality known as **continuous**. This continuity would be like following a road rather than stepping stones.

SSG **EXS. 3–5**

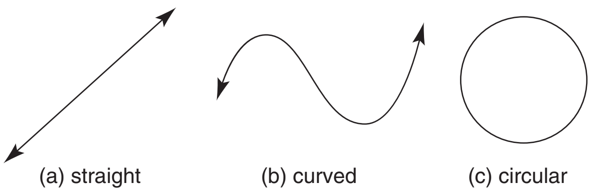


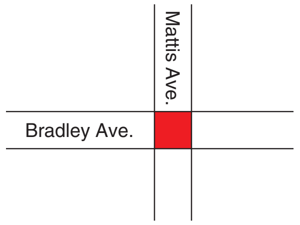
Figure P.6

INTERSECTION AND UNION OF SETS

DEFINITION

The set of elements that are common to (belong to both) sets P and Q is known as the **intersection** of P and Q. The symbol for this intersection is $P \cap Q$.

Geometry in the Real World



In the real world, the common part of two roads is known as their intersection.

EXAMPLE 3

Where $A = \{1,2,3,4\}$ and $B = \{2,4,6,8\}$, find the intersection $A \cap B$. Also, find $N\{A \cap B\}$.

SOLUTION Because 2 and 4 are the only elements that appear in both sets, $A \cap B = \{2,4\}$. Because $A \cap B$ contains 2 elements, we say that $N\{A \cap B\} = 2$.

In Figure P.7, the intersection of $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ is point E; that is, $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = E$.

A set containing no elements is called the **empty set** and is represented by the symbol $\emptyset$; for instance, the set of counting numbers between 4 and 5 is the empty set. Also, the two lines $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ in Figure P.8 have no points in common; thus, $\overleftrightarrow{AB} \cap \overleftrightarrow{CD} = \emptyset$.

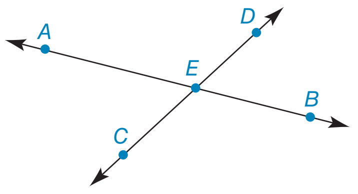


Figure P. 7

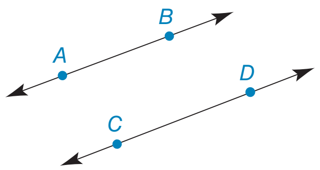


Figure P. 8

Two sets that have no elements in common are also known as **disjoint sets**. For example, the sets $C = \{1,3\}$ and $D = \{2,4\}$ are disjoint sets.

Warning

We will describe a set by one of the following: empty, finite, or infinite. For clarification, a finite set has a counting number $\{1,2,3,4,\ldots\}$ of elements while the empty set has 0 elements.

DEFINITION

The set of elements that are in set P, set Q, or both sets P and Q is known as the **union** of P and Q. The symbol for the union of sets P and Q is $P \cup Q$.

EXAMPLE 4

Where $A = \{1,2,3,4\}$ and $B = \{2,4,6,8\}$, find the union $A \cup B$. Also, find $N\{A \cup B\}$.

SOLUTION Combining all elements in A and B, we have $A \cup B = \{1,2,3,4,6,8\}$. Because $A \cup B$ contains 6 elements, we have $N\{A \cup B\} = 6$.

SSG **EXS. 5–8**

In geometry, we will define an **angle** as the union of two rays with a common endpoint. In Figure P.9, $\overrightarrow{BA}$ and $\overrightarrow{BC}$ are known as rays; $\overrightarrow{BA} \cup \overrightarrow{BC} = \angle ABC$, where $\angle ABC$ is read "angle ABC" and traces a path from A to B, and then from B to C.

The fact that every angle has two sides can be expressed by writing $N\{\text{sides of an angle}\} = 2$. Later we will measure an angle by using an instrument called the protractor.

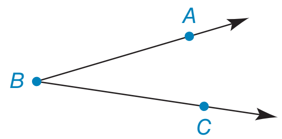


Figure P. 9

SETS AND VENN DIAGRAMS

Sets of objects are sometimes represented by geometric figures such as circles or rectangles. These figures are known as **Venn Diagrams** in honor of their creator, John Venn, an Englishman who lived from 1834 A.D. to 1923 A.D. If we represent sets by A and B, there are several possible relationships, as shown in Figure P.10. When the figures representing the two sets intersect, we sometimes say that the Venn Diagrams overlap; see Figure P.10(b).

Two sets that have exactly the same elements are known as **equivalent sets**. The two sets $E = \{2,4,6,8\}$ and $F = \{\text{even numbers between 1 and 9}\}$ are equivalent, written $E = F$. When the two sets are equivalent, only one set is shown (but labeled twice), as in Figure P.10(d).

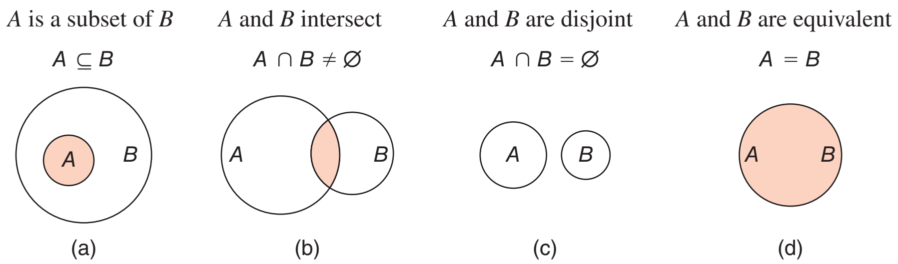


Figure P.10

EXAMPLE 5

Which relationship (subset, intersect, disjoint, equivalent) exists between these sets?

a) $A = \{1,2,3\}$; $B = \{\text{counting numbers less than } 4\}$
b) $C = \{2,4,6,8\}$; $D = \{\text{all even numbers}\}$
c) $E = \{2,3,5,7,11\}$; $F = \{\text{single digit counting numbers}\}$
d) $G = \{\text{all even numbers}\}$; $H = \{\text{all odd numbers}\}$

SOLUTION

a) Because $B = \{1,2,3\}$, sets A and B are equivalent; that is, $A = B$.
b) Because $D = \{\ldots,-6,-4,-2,0,2,4,6,8,10,\ldots\}$, C is a subset of D; that is, $C \subseteq D$.
c) Because $F = \{1,2,3,4,\ldots,9\}$, $E \cap F = \{2,3,5,7\}$; that is, E and F intersect.
d) Because no even number is odd, $G \cap H = \varnothing$; these sets are disjoint.

SSG **EXS. 9, 10**

In some discussions, the term **universe** is used to describe a set that contains all of the elements and subsets under discussion. In the form of a Venn Diagram, all subsets appear within the universe (or universal set). In Figure P.11, let $D = \{\text{dogs}\}$, while $P = \{\text{pomeranians}\}$, $C = \{\text{collies}\}$, and $B = \{\text{beagles}\}$. In Figure P.11, D is the universal set containing the disjoint subsets P, C, and B.

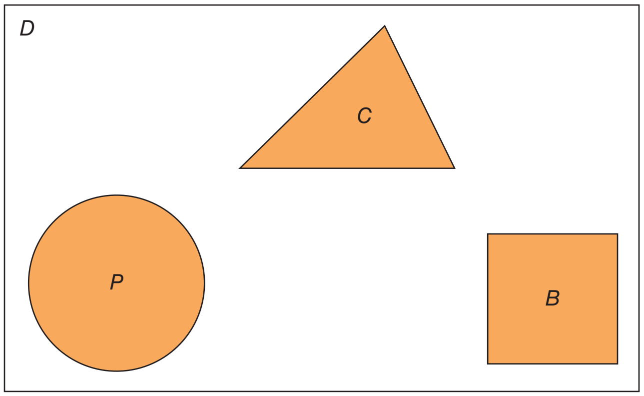


Figure P.11

In geometry, we will use the notion of a universe many times, but perhaps without calling so much attention to it. For instance, the universe implied in Chapter 3 will be $T = \{\text{triangles}\}$. A triangle, as you may very well know, has 3 sides that are line segments. In that chapter, many subsets will be considered: triangles with 2 sides of equal length, triangles with 3 sides of equal length, triangles in which one of the angles formed is known as a right angle, and so on. In Chapter 4, we will focus upon $Q = \{\text{quadrilaterals}\}$, figures with 4 line segment sides; some of these are squares and others rectangles. Where P is a subset of the universe U, we define a related concept.

DEFINITION

The **complement** of set P, a subset of universe U, is the set that contains the elements of U that are not in P. The symbol $\sim P$ represents the complement of P.

SSG **EXS. 11, 12**

The shaded region in Figure P.12 represents the complement of P. Informally, the complement is the portion of U not in P.

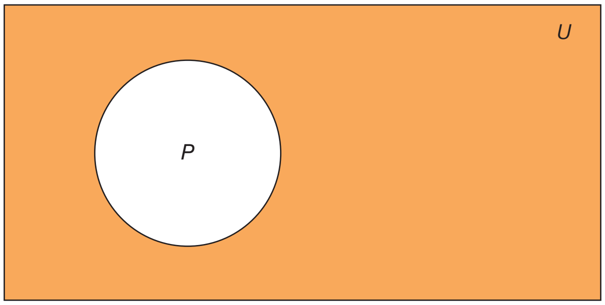


Figure P.12

Exercises P.1

1. Is the set named empty, finite, or infinite?
   a) N = {counting numbers}
   b) V = {vowels}
   c) P = {points in the intersection of two disjoint sets}

2. Is the set named empty, finite, or infinite?
   a) L = {points on a line}
   b) B = {baseballs used in a major league game}
   c) N = {negative numbers that are larger than 5}

3. Where possible, use the symbol $\subseteq$ to join the two sets.
   a) V = {vowels}; L = {letters in the alphabet}
   b) $A = \{1,2,3,4\}$; $B = \{2,4,6,8\}$

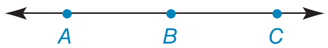
4. Where possible, use the symbol $\subseteq$ to join the two sets.
   a) P = {all pages in this book};
      T = {page 10 of this book}
   b) P = {points on a circular path P};
      E = {points A and B that lie on the circular path P}

5. Which type of path (straight, curved, circular, or scattered) is determined by:
   a) a roller coaster?
   b) a pencil?
   c) a carousel?
   d) darts thrown at a dartboard?

6. Which type of path (straight, curved, circular, or scattered) is determined by:
   a) pepper sprinkled on a steak?
   b) a Ferris wheel?
   c) a ruler?
   d) the path of a snake?

7. Let A, B, and C lie on a straight line as shown near Exercise 8. Classify these claims as true or false.
   a) B lies between A and C.
   b) $\overrightarrow{AB}$ is a ray with endpoint B.

8. Let A, B, and C lie on a straight line as shown. Classify these claims as true or false.
   a) There is no point that lies between A and B.
   b) $\overline{AB}$ has endpoints A and B.



Exercises 7, 8

9. Given $A = \{1,2,3,4\}$, $B = \{2,4,6,8\}$, and $C = \{1,3,5,7,9\}$. Find:
   a) $A \cap B$
   b) $B \cup C$
   c) $(A \cap B) \cap (B \cup C)$

10. Consider sets A, B, and C from Exercise 9. Find:
    a) $A \cup B$
    b) $B \cap C$
    c) $(A \cup B) \cap (A \cup C)$

11. For the sets given in Exercise 9, is there a "distributive relationship for union with respect to intersection"? That is, does $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$?

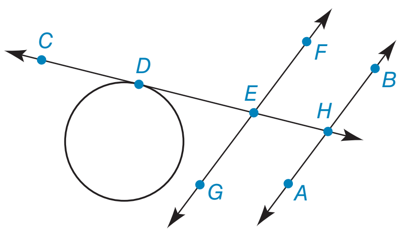
12. For the sets given in Exercise 9, is there a "distributive relationship for intersection with respect to union"? That is, does $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$?

For Exercises 13 to 16, consider the figure with the circle and lines $\overleftrightarrow{AB}$, $\overleftrightarrow{CE}$, and $\overleftrightarrow{GF}$. Classify the following claims as true or false.

13. The circle and $\overleftrightarrow{AB}$ intersect in two points.

14. $\overleftrightarrow{CD} \cap \overleftrightarrow{GF} = E$

15. Two lines must intersect in one point.



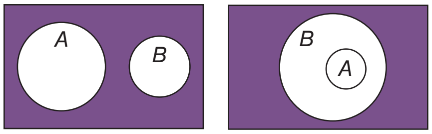
Exercises 13–16

16. The circle and $\overleftrightarrow{CE}$ intersect in point D.

17. Use the angle symbol ($\angle$) and three letters to name the angle formed by the two rays $\overrightarrow{CD}$ and $\overrightarrow{CE}$ (rays not shown).

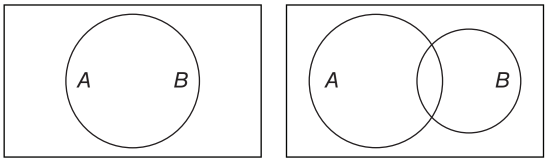
18. Use the angle symbol ($\angle$) and three letters to name the angle formed by the two rays $\overrightarrow{EF}$ and $\overrightarrow{EG}$ (rays not shown).

19. What relationship (subset, intersect, disjoint, or equivalent) can be used to characterize the two sets shown in the Venn Diagram?



(a) (b)

20. What relationship (subset, intersect, disjoint, or equivalent) can be used to characterize the two sets shown in the Venn Diagram?



(a) (b)

21. If $A = \{1,2,3,4\}$ and $B = \{2,4,6,8\}$, find:
a) $N\{A\}$
b) $N\{B\}$
c) $N(A \cap B)$
d) $N(A \cup B)$

22. If $A = \{\text{vowels}\}$ and $B = \{\text{letters in the word "spare"}\}$, find:
a) $N\{A\}$
b) $N\{B\}$
c) $N(A \cap B)$
d) $N(A \cup B)$

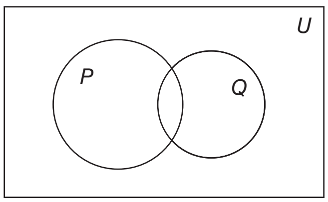
23. Find a universe for the set {algebra, geometry, trigonometry, calculus}.

24. Find a universe for the set {apples, bananas, pears, peaches, strawberries}.

25. Where $P = \{1,3,5,7,9\}$ and $U = \{1,2,3,4,5,6,7,8,9\}$, find $\sim P$.

26. Where $C = \{\text{consonants}\}$ and $U = \{\text{26 letters of the alphabet}\}$, find $\sim C$.

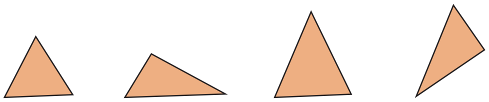
27. With P and Q subsets of U (as shown), shade $\sim(P \cup Q)$.



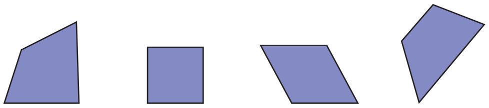
Exercises 27, 28

28. With P and Q subsets of U (as shown), shade $\sim(P \cap Q)$.

29. The figures below are triangles. Find: $N\{\text{sides in a triangle}\}$.



30. The figures below are quadrilaterals. Find: $N\{\text{sides in a quadrilateral}\}$.



31. If $N\{A\} = 3$ and $N\{B\} = 5$, what is:
a) the smallest number possible for $N\{A \cap B\}$?
b) the largest number possible for $N\{A \cup B\}$?

32. If $N\{A\} = 3$ and $N\{B\} = 5$, what is:
a) the largest number possible for $N\{A \cap B\}$?
b) the smallest number possible for $N\{A \cup B\}$?

Exercises 33–38 are based on the principle $N(A \cup B) = N\{A\} + N\{B\} - N(A \cap B)$.

33. If $N\{A\} = 7$ and $N\{B\} = 7$ and $N(A \cup B) = 7$, how are A and B related?

34. If $N\{A\} = 7$ and $N\{B\} = 3$ and $N(A \cup B) = 10$, how are A and B related?

35. An interview of 50 women in a residential subdivision reveals that 21 are gardeners while 32 are avid readers. How many are both gardeners and avid readers?

36. At a liberal arts school, a survey of 100 students indicates that 68 enjoy their English class while 47 enjoy their mathematics class. How many of these students enjoy both their English class and their mathematics class?

37. Alekzio, a librarian at the city library, is reshelving books. He replaces 47 mysteries and 69 romance novels; of these, 32 were cross-listed as both mystery and romance. How many books did Alekzio reshelve in all?

38. At "It's a Small World," dance classes and gymnastics classes are offered to preschool-age children. For the fall classes, 65 preschoolers are enrolled in dance classes and 58 are enrolled in gymnastics classes. Of those students, 26 were enrolled in both dance and gymnastics classes. How many total students were enrolled?

P.2 Statements and Reasoning

KEY CONCEPTS

Statement
Variable
Negation
Compound Statement
Conjunction
Disjunction
Conditional Statement (Implication)
Hypothesis
Conclusion
Reasoning
Intuition
Induction
Deduction
Valid and Invalid Arguments
Law of Detachment
Counterexample

STATEMENTS

DEFINITION

A **statement** is a set of words and/or symbols that collectively make a claim that can be classified as true or false.

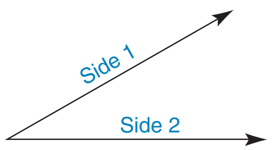


Figure P.13

EXAMPLE 1

Classify each of the following as a true statement, a false statement, or neither.

1. $4 + 3 = 7$
2. An angle has two sides. (See Figure P.13.)
3. Robert E. Lee played shortstop for the Yankees.
4. $7 < 3$ (This is read "7 is less than 3.")
5. Look out!

SOLUTION 1 and 2 are true statements; 3 and 4 are false statements; 5 is not a statement.

Some statements contain one or more *variables*; a **variable** is a letter that represents a number. The claim "$x + 5 = 6$" is called an *open sentence* or *open statement* because it can be classified as true or false, depending on the replacement value of x. For instance, $x + 5 = 6$ is true if $x = 1$; for x not equal to 1, $x + 5 = 6$ is false. Some statements containing variables are classified as true because they are true for all replacements. Consider the Commutative Property of Addition, usually stated in the form $a + b = b + a$. In words, this property states that the same result is obtained when two numbers are added in either order; for instance, when $a = 4$ and $b = 7$, it follows that $4 + 7 = 7 + 4$.

The **negation** of a given statement P makes a claim opposite that of the original statement. If the given statement is true, its negation is false, and vice versa. If P is a statement, we use $\sim P$ (which is read "not P") to indicate its negation.

EXAMPLE 2

Give the negation of each statement.

a) $4 + 3 = 7$ **b)** All fish can swim.

SOLUTION

a) $4 + 3 \neq 7$ ($\neq$ means "is not equal to.")
b) Some fish cannot swim. (To negate "All fish can swim," we say that at least one fish cannot swim.)

A **compound** statement is formed by combining other statements used as "building blocks." In such cases, we may use letters such as P and Q to represent simple statements.

For example, the letter P may refer to the true statement "$4 + 3 = 7$," and the letter Q to the false statement "Babe Ruth was a U.S. president." The statement "$4 + 3 = 7$ *and* Babe Ruth was a U.S. president" has the form P and Q, and is known as the **conjunction** of P and Q. The statement "$4 + 3 = 7$ *or* Babe Ruth was a U.S. president" has the form P or Q, and is known as the **disjunction** of statement P and statement Q. A conjunction is true only when P and Q are *both* true. A disjunction is false only when P and Q are *both* false. See Tables P.1 and P.2.

TABLE P.1
The Conjunction

P	Q	P and Q
T	T	T
T	F	F
F	T	F
F	F	F

TABLE P.2
The Disjunction

P	Q	P or Q
T	T	T
T	F	T
F	T	T
F	F	F

EXAMPLE 3

Assume that statement P and statement Q are both true.

P: $4 + 3 = 7$
Q: An angle has two sides.

Classify the following statements as true or false.

1. $4 + 3 \neq 7$ and an angle has two sides.
2. $4 + 3 \neq 7$ or an angle has two sides.

SOLUTION Statement 1 is false because the conjunction has the form "F and T." Statement 2 is true because the disjunction has the form "F or T."

The statement "If P, then Q," known as a **conditional statement** (or **implication**), is classified as true or false as a whole. A statement of this form can be written in equivalent forms; for instance, the conditional statement, "If an angle is a right angle, then it measures 90 degrees" is equivalent to the statement, "All right angles measure 90 degrees."

EXAMPLE 4

Classify each conditional statement as true or false.

1. If an animal is a fish, then it can swim. (States, "All fish can swim.")
2. If two sides of a triangle are equal in length, then two angles of the triangle are equal in measure. (See Figure P.14.)

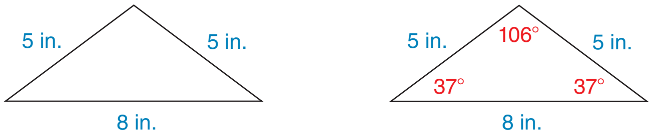


Figure P.14

3. If Wendell studies, then he will receive an A on the test.

SOLUTION Statements 1 and 2 are true. Statement 3 is false; Wendell may study yet not receive an A.

In the conditional statement "If P, then Q," P is the **hypothesis** and Q is the **conclusion**. In statement 2 of Example 4, we have

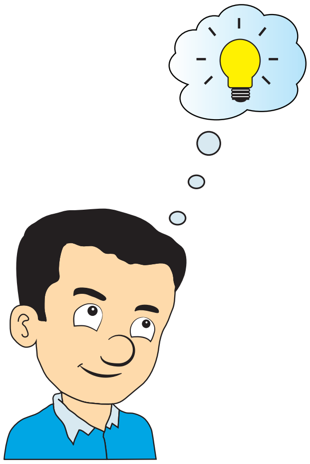
Hypothesis: Two sides of a triangle are equal in length.

Conclusion: Two angles of the triangle are equal in measure.

For the true statement "If P, then Q," the hypothetical situation described in P implies the conclusion described in Q. This type of statement is often used in reasoning, so we turn our attention to this matter.

SSG EXS. 1–7

Geometry in the Real World



In a cartoon, the character having the "bright idea" (using intuition) is shown with a light bulb next to his or her head.

REASONING

Success in the study of geometry requires vocabulary development, attention to detail and order, supporting claims, and thinking. **Reasoning** is a process based on experience and principles that leads to a conclusion. The following types of reasoning are used to develop mathematical principles.

1.	Intuition	an inspiration leading to the statement of a theory
2.	Induction	an organized effort to test and validate the theory
3.	Deduction	a formal argument that proves the tested theory

◆ Intuition

We are often inspired to think and say, "It occurs to me that. . . ." With **intuition**, a sudden insight allows one to make a statement without applying any formal reasoning. When intuition is used, we sometimes err by "jumping" to conclusions.

EXAMPLE 5

Figure P.15 is called a *regular pentagon* because its five sides have equal lengths and its five interior angles have equal measures. What do you suspect is true of the lengths of the dashed parts of lines from B to E and from B to D?

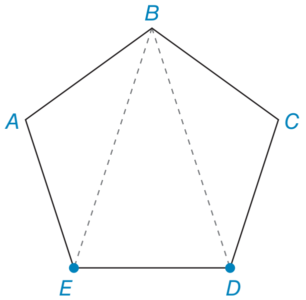


Figure P.15

SOLUTION Intuition suggests that the lengths of the dashed parts of lines (known as *diagonals* of the pentagon) are the same.

NOTE 1: Using induction (and a *ruler*), we can verify that this claim is true. We will discuss measurement with the ruler in more detail in Section P.3.

NOTE 2: Using methods found in Chapter 3, we could use deduction to prove that the two diagonals do indeed have the same length. ■

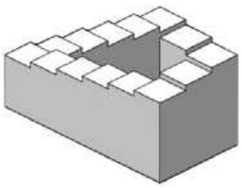
The role intuition plays in formulating mathematical thoughts is truly significant. But to have an idea is not enough! Testing a theory may lead to a revision of the theory or even to its total rejection. If a theory stands up to testing, it moves one step closer to becoming mathematical law.

◆ Induction

We often use specific observations and experiments to draw a general conclusion. This type of reasoning is called **induction**. As you would expect, the observation/experimentation process is common in laboratory and clinical settings. Chemists, physicists, doctors, psychologists, meteorologists (weather forecasters), and many others use collected data as a basis for drawing conclusions. In our study of geometry, the inductive process generally has us use a ruler or a *protractor* (to measure angles).

Discover

An optical illusion known as "Penrose stairs" is shown below. Although common sense correctly concludes that no such stairs can be constructed, what unusual quality appears to be true of the stairs drawn?



ANSWER

The stairs constantly rise or descend.

EXAMPLE 6

While in a grocery store, you examine several 6-oz cartons of yogurt. Although the flavors and brands differ, each carton is priced at 95 cents. What do you conclude?

CONCLUSION Every 6-oz carton of yogurt in the store costs 95 cents. ■

EXAMPLE 7

In a geometry class, you have been asked to measure the three interior angles of each triangle in Figure P.16. You discover that triangles I, II, and IV have two angles (as marked) that have equal measures. What may you conclude?

CONCLUSION The triangles that have two sides of equal length also have two angles of equal measure.

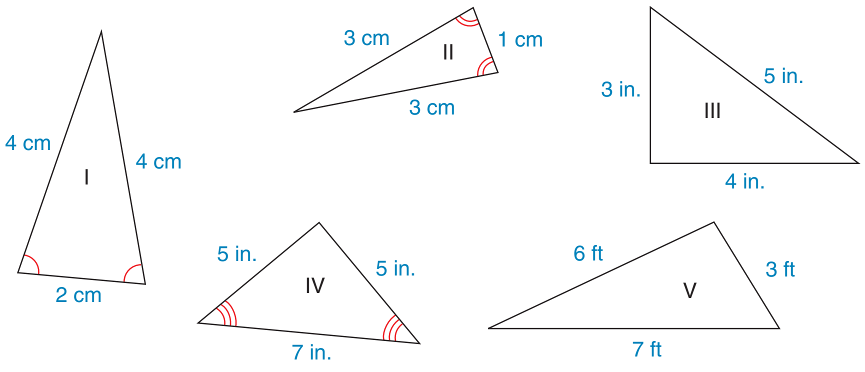


Figure P.16

NOTE: The protractor, used to support the conclusion above, will be discussed in Section P.3.

◆ Deduction

DEFINITION

Deduction is the type of reasoning in which the knowledge and acceptance of selected assumptions guarantee the truth of a particular conclusion.

In Example 8, we illustrate a **valid argument**, a form of deductive reasoning used frequently in the development of geometry. In this form, at least two statements are treated as facts; these assumptions are called the *premises* of the argument. On the basis of the premises, a particular *conclusion* must follow. This form of deduction is called the **Law of Detachment**.

EXAMPLE 8

If you accept the following statements 1 and 2 as true, what must you conclude?

1. If a student plays on the Rockville High School boys' varsity basketball team, then he is a talented athlete.
2. Todd plays on the Rockville High School boys' varsity basketball team.

CONCLUSION Todd is a talented athlete.

To more easily recognize this pattern for deductive reasoning, we use letters to represent statements in the following generalization.

LAW OF DETACHMENT

Let P and Q represent simple statements, and assume that statements 1 and 2 are true. Then a valid argument having conclusion C has the form

1. If P, then Q
2. P } **premises**

C. $\therefore Q$ } **conclusion**

NOTE: The symbol $\therefore$ means "therefore."

In the preceding form, the statement "If P, then Q" is often read "P implies Q." That is, when P is known to be true, Q must follow.

EXAMPLE 9

Is the following argument valid? Assume that premises 1 and 2 are true.

1. If it is raining, then Tim will stay in the house.
2. It is raining.

C. $\therefore$ Tim will stay in the house.

CONCLUSION The argument is valid because the form of the argument is

1. If P, then Q
2. P

C. $\therefore Q$

with P = "It is raining," and Q = "Tim will stay in the house."

EXAMPLE 10

Is the following argument valid? Assume that premises 1 and 2 are true.

1. If a man lives in London, then he lives in England.
2. William lives in England.

C. $\therefore$ William lives in London.

CONCLUSION The argument is not valid. Here, P = "A man lives in London," and Q = "A man lives in England." Thus, the form of this argument is

1. If P, then Q
2. Q

C. $\therefore P$

To represent a valid argument, the Law of Detachment would require that the first statement has the form "If Q, then P." Even though statement Q is true, it does not enable us to draw a valid conclusion about P. Thus, we have an **invalid argument**. Of course, if William lives in England, he *might* live in London; but he might instead live in Liverpool, Manchester, Coventry, or any of countless other places in England. Each of these possibilities is a **counterexample** disproving the validity of the argument. Remember that deductive reasoning is concerned with reaching conclusions that *must be true*, given the truth of the premises.

Warning

In the box, the argument on the left is valid and patterned after Example 9. The argument on the right is invalid; this form was given in Example 10.

VALID ARGUMENT	INVALID ARGUMENT
1. If P, then Q	**1.** If P, then Q
2. P	**2.** Q
C. $\therefore Q$	C. $\therefore P$

We will use deductive reasoning throughout our work in geometry. For example, suppose that you know these two facts:

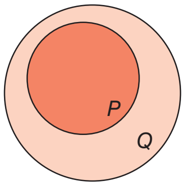
SSG EXS. 8–12

1. If an angle is a right angle, then it measures 90°.
2. Angle A is a right angle.

Because the form found in statements 1 and 2 matches the form of the valid argument, you may draw the following conclusion.

C. Angle A measures 90°.

VENN DIAGRAMS



If P, then Q.

Figure P.17

As described in Section P.1, sets of objects are often represented by *Venn Diagrams*. In a Venn Diagram, each set is represented by a closed (bounded) figure such as a circle or rectangle. If statements P and Q of the conditional statement "If P, then Q" are represented by sets of objects P and Q, respectively, then the Law of Detachment can be justified by a geometric argument. When a Venn Diagram is used to represent the statement "If P, then Q," it is absolutely necessary that circle P lies in circle Q; that is, P is a *subset* of Q. (See Figure P.17.)

EXAMPLE 11

Use Venn Diagrams to verify Example 8.

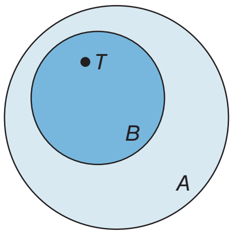


Figure P.18

SOLUTION Let B = students on the Rockville High varsity boys' basketball team. Let A = people who are talented athletes.

To represent the statement "If a basketball player (B), then a talented athlete (A)," we show B within A. In Figure P.18 we use point T to represent Todd, a person on the basketball team (T in B). With point T also in circle A, we conclude that "Todd is a talented athlete." ■

The statement "If P, then Q" is sometimes expressed in the form "All P are Q." For instance, the conditional statement of Examples 8 and 11 can be written "All Rockville High School basketball players are talented athletes." Venn Diagrams can also be used to demonstrate that the argument of Example 10 is not valid. To show the invalidity of the argument in Example 10, one must show that an object in Q may *not* lie in circle P. (See Figure P.17.)

In Exercises 33–38 of Section P.1, we applied the following principle:

$$N\{P \cup Q\} = N\{P\} + N\{Q\} - N\{P \cap Q\}.$$

Consider the Discover activity at the left.

Discover

in the St. Louis area, an interview of 100 sports enthusiasts shows that 74 support the Cardinals baseball team and 58 support the Blues hockey team. All of those interviewed support one team or the other or both. How many support both teams?

ANSWER

32; $74 + 58 - 100$

We now use a reasoning approach with Venn Diagrams in order to verify this principle. In this situation, we assume that P and Q have elements in common; however, the statement above remains true even if P and Q are disjoint sets.

In Figure P.19, the number of elements shown in the light blue region is x, the number of elements shown in the gold region is z, and the number of elements shown in the red region is y.

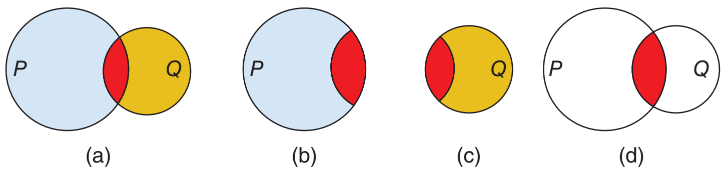


Figure P.19

Considering Figure P.19, we find that $N\{P\} = x + y$, $N\{Q\} = y + z$, and $N\{P \cap Q\} = y$. We can now verify the statement $N\{P \cup Q\} = N\{P\} + N\{Q\} - N\{P \cap Q\}$.

$$N\{P \cup Q\} = (x + y) + (y + z) - y$$
$$\text{or } x + 2y + z - y \text{ or } x + y + z,$$

SSG **EXS. 13–15**

which is the number of elements shown in $P \cup Q$.

Exercises P.2

In Exercises 1 and 2, which sentences are statements? If a sentence is a statement, classify it as true or false.

1. a) Where do you live?
b) $4 + 7 \neq 5$.
c) Washington was the first U.S. president.
d) $x + 3 = 7$ when $x = 5$.

2. a) Chicago is located in the state of Illinois.
b) Get out of here!
c) $x < 6$ (read as "x is less than 6") when $x = 10$.
d) Babe Ruth is remembered as a great football player.

In Exercises 3 and 4, give the negation of each statement.

3. a) Christopher Columbus crossed the Atlantic Ocean.
b) All jokes are funny.

4. a) No one likes me.
b) Angle 1 is a right angle.

In Exercises 5 to 10, classify each statement as simple, conditional, a conjunction, or a disjunction.

5. If Alice plays, the volleyball team will win.

6. Alice played and the team won.

7. The first-place trophy is beautiful.

8. An integer is odd or it is even.

9. Matthew is playing shortstop.

10. You will be in trouble if you don't change your ways.

In Exercises 11 to 18, state the hypothesis and the conclusion of each statement.

11. If you go to the game, then you will have a great time.

12. If two chords of a circle have equal lengths, then the arcs of the chords are congruent.

13. If the diagonals of a parallelogram are perpendicular, then the parallelogram is a rhombus.

14. If $\frac{a}{b} = \frac{c}{d}$, where $b \neq 0$ and $d \neq 0$, then $a \cdot d = b \cdot c$.

15. Corresponding angles are congruent if two parallel lines are cut by a transversal.

16. Vertical angles are congruent when two lines intersect.

17. All squares are rectangles.

18. Base angles of an isosceles triangle are congruent.

In Exercises 19 to 24, classify each statement as true or false.

19. If a number is divisible by 6, then it is divisible by 3.

20. Rain is wet and snow is cold.

21. Rain is wet or snow is cold.

22. If Jim lives in Idaho, then he lives in Boise.

23. Triangles are round or circles are square.

24. Triangles are square or circles are round.

In Exercises 25 to 32, name the type of reasoning (if any) used.

25. While participating in an Easter egg hunt, Sarah notices that each of the seven eggs she has found is numbered. Sarah concludes that all eggs used for the hunt are numbered.

26. You walk into your geometry class, look at the teacher, and conclude that you will have a quiz today.

27. Lucy knows the rule "If a number is added to each side of an equation, then the new equation has the same solution set as the given equation." Given the equation $x - 5 = 7$, Lucy concludes that $x = 12$.

28. You believe that "Anyone who plays major league baseball is a talented athlete." Knowing that Duane Gibson has just been called up to the major leagues, you conclude that Duane Gibson is a talented athlete.

29. As a handcuffed man is brought into the police station, you glance at him and say to your friend, "That fellow looks guilty to me."

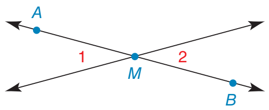
30. While judging a science fair project, Mr. Cange finds that each of the first 5 projects is outstanding and concludes that all 10 will be outstanding.

31. You know the rule "If a person lives in the Santa Rosa Junior College district, then he or she will receive a tuition break at Santa Rosa." Emma tells you that she has received a tuition break. You conclude that she resides in the Santa Rosa Junior College district.

32. Before Mrs. Gibson enters the doctor's waiting room, she concludes that it will be a long wait.

In Exercises 33 to 36, use intuition to state a conclusion.

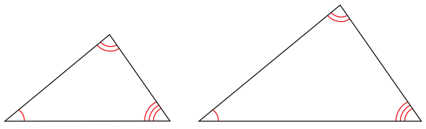
33. You are told that the opposite angles formed when two lines cross are **vertical angles**. In the figure, angles 1 and 2 are vertical angles. Conclusion?



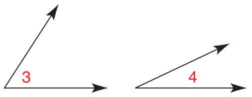
Exercises 33, 34

34. In the figure, point M is called the **midpoint** of line segment AB. Conclusion?

35. The two triangles shown are **similar** to each other. Conclusion?



36. Observe (but do not measure) the following angles. Conclusion?



In Exercises 37 to 40, use induction to state a conclusion.

37. Several movies directed by Lawrence Garrison have won Academy Awards, and many others have received nominations. His latest work, *A Prisoner of Society*, is to be released next week. Conclusion?

38. On Monday, Matt says to you, "Andy hit his little sister at school today." On Tuesday, Matt informs you, "Andy threw his math book into the wastebasket during class." On Wednesday, Matt tells you, "Because Andy was throwing peas in the school cafeteria, he was sent to the principal's office." Conclusion?

39. While searching for a classroom, Tom stopped at an instructor's office to ask directions. On the office bookshelves are books titled *Intermediate Algebra, Calculus, Modern Geometry, Linear Algebra*, and *Differential Equations*. Conclusion?

40. At a friend's house, you see several food items, including apples, pears, grapes, oranges, and bananas. Conclusion?

In Exercises 41 to 50, use deduction to state a conclusion, if possible.

41. If the sum of the measures of two angles is 90°, then these angles are called "complementary." Angle 1 measures 27° and angle 2 measures 63°. Conclusion?

42. If a person attends college, then he or she will be a success in life. Kathy Jones attends Dade County Community College. Conclusion?

43. All mathematics teachers have a strange sense of humor. Alex is a mathematics teacher. Conclusion?

44. All mathematics teachers have a strange sense of humor. Alex has a strange sense of humor. Conclusion?

45. If Stewart Powers is elected president, then every school will have internet access. Every school has internet access. Conclusion?

46. If Tabby is meowing, then she is hungry. Tabby is hungry. Conclusion?

47. If a person is involved in politics, then that person will be in the public eye. June Jesse has been elected to the Missouri state senate. Conclusion?

48. If a student is enrolled in a literature course, then he or she will work very hard. Bram Spiegel digs ditches by hand six days a week. Conclusion?

49. If a person is rich and famous, then he or she is happy. Marilyn is wealthy and well known. Conclusion?

50. If you study hard and hire a tutor, then you will make an A in this course. You make an A in this course. Conclusion?

In Exercises 51 to 54, use Venn Diagrams to determine whether the argument is valid or not valid.

51. 1) If an animal is a cat, then it makes a "meow" sound.
2) Tipper is a cat.
C) Then Tipper makes a "meow" sound.

52. 1) If an animal is a cat, then it makes a "meow" sound.
2) Tipper makes a "meow" sound.
C) Then Tipper is a cat.

53. 1) All Boy Scouts serve the United States of America.
2) Sean serves the United States of America.
C) Sean is a Boy Scout.

54. 1) All Boy Scouts serve the United States of America.
2) Sean is a Boy Scout.
C) Sean serves the United States of America.

In Exercises 55 and 56, P is a true statement, while Q and R are false statements. Classify each of the following statements as true or false.

55. a) (P and Q) or $\sim R$
b) (P or Q) and $\sim R$

56. a) (P and Q) or R
b) (P or Q) and R

P.3 Informal Geometry and Measurement

KEY CONCEPTS

Point	Midpoint	Trisect	Compass
Line	Congruent	Straight angle	Constructions
Collinear Points	Protractor	Right Angle	Circle
Vertex	Parallel	Intersect	Arc
Line Segment	Bisect	Perpendicular	Radius

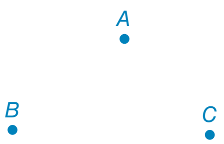
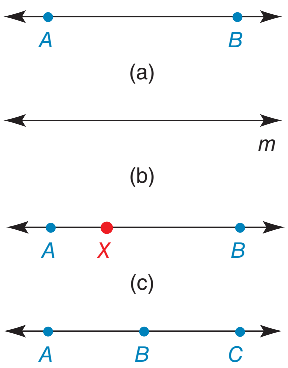


Figure P.20



(a) (b) (c) (d)

Figure P.21

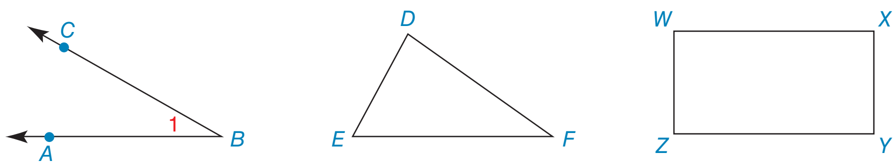
In geometry, the terms *point*, *line*, and *plane* are described but not defined. Other concepts that are accepted intuitively, but never defined, include the *straightness* of a line, the *flatness* of a plane, the notion that a point on a line lies *between* two other points on the line, and the notion that a point lies in the *interior* or *exterior* of an angle. Some of the terms found in this section are formally defined in later sections of Chapter 1. The following are descriptions of some of the undefined terms.

A **point**, which is represented by a dot, has location but not size; that is, a point has no dimensions. An uppercase italic letter is used to name a point. Figure P.20 shows points *A*, *B*, and *C*.

The second undefined geometric term is **line**. A line is an infinite set of points. Given any two points on a line, there is always a point that lies between them on that line. Lines have a quality of "straightness" that is not defined but assumed. Given several points on a line, these points form a straight path. Whereas a point has no dimensions, a line is one-dimensional; that is, the distance between any two points on a given line can be measured. Line *AB*, represented symbolically by $\overleftrightarrow{AB}$, extends infinitely far in opposite directions, as suggested by the arrows on the line. A line may also be represented by a single lowercase letter. Figure P.21(a) and (b) show the lines *AB* and *m*. When a lowercase letter is used to name a line, the line symbol is omitted; that is, $\overleftrightarrow{AB}$ and *m* can name the same line.

Note the position of point *X* on $\overleftrightarrow{AB}$ in Figure P.21(c). When three points such as *A*, *X*, and *B* are on the same line, they are said to be **collinear**. In the order shown, which is symbolized *A*-*X*-*B* or *B*-*X*-*A*, point *X* is said to be *between A* and *B*. When a drawing is not provided, the notation *A*-*B*-*C* means that these points are collinear, with *B between A* and *C*. When a drawing is provided, we assume that all points in the drawing that appear to be collinear *are* collinear, *unless otherwise stated*. Figure P.21(d) shows that *A*, *B*, and *C* are collinear; in Figure P.22(a), points *A*, *B*, and *C* are *noncollinear*.

At this time, we informally introduce some terms that will be formally defined later. You have probably encountered the terms *angle*, *triangle*, and *rectangle* many times. An example of each is shown in Figure P.22.



Angle *ABC* (a) Triangle *DEF* (b) Rectangle *WXYZ* (c)

Figure P.22

Using symbols, we refer to Figure P.22(a), (b), and (c) as $\angle ABC$, $\triangle DEF$, and ▭*WXYZ*, respectively. Some caution must be used in naming figures; although the angle in Figure P.22(a) can be called $\angle CBA$, it is incorrect to describe the angle as $\angle ACB$ because that order implies a path from point *A* to point *C* to point *B* . . . a different angle! In $\angle ABC$, the point *B* at which the sides meet is called the **vertex** of the angle. Because there is no confusion regarding the angle described, $\angle ABC$ is also known as $\angle B$ (using only the vertex) or as $\angle 1$. The points D, E, and F at which the sides of $\triangle DEF$ (also called $\triangle DFE$, $\triangle EFD$, etc.) meet are called the *vertices* (plural of *vertex*) of the triangle. Similarly, W, X, Y, and Z are the vertices of the rectangle; the vertices are named in an order that traces the rectangle.

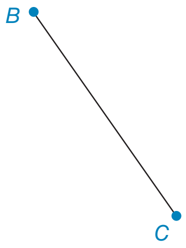


Figure P.23

A **line segment** is part of a line. It consists of two distinct points on the line and all points between them. (See Figure P.23.) Using symbols, we indicate the line segment by $\overline{BC}$; note that $\overline{BC}$ is a set of points but is not a number. We use BC (omitting the segment symbol) to indicate the *length* of this line segment; thus, BC is a number. The sides of a triangle or rectangle are line segments.

EXAMPLE 1

Can the rectangle in Figure P.22(c) be named **a)** $\square XYZW$? **b)** $\square WYXZ$?

SOLUTION

a) Yes, because the points taken in this order trace the figure.
b) No; for example, $\overline{WY}$ is not a side of the rectangle.

MEASURING LINE SEGMENTS

Discover

In converting from U.S. units to the metric system, a known conversion is the fact that 1 inch $\approx$ 2.54 cm. What is the "cm" equivalent of 3.7 inches?

ANSWER
9.4 cm

The instrument used to measure a line segment is a scaled straightedge such as a *ruler*, a *yardstick*, or a *meter stick*. Line segment RS ($\overline{RS}$ in symbols) in Figure P.24 measures 5 centimeters. Because we express the length of $\overline{RS}$ by RS (with no bar), we write $RS = 5$ cm.

To find the length of a line segment using a ruler:

1. Place the ruler so that "0" corresponds to one endpoint of the line segment.
2. Read the length of the line segment by reading the number at the remaining endpoint of the line segment.

Because manufactured measuring devices such as the ruler, yardstick, and meter stick may lack perfection or be misread, there is a margin of error each time one is used. In Figure P.24, for instance, RS may actually measure 5.02 cm (and that could be rounded from 5.023 cm, etc.). Measurements are approximate, not perfect.

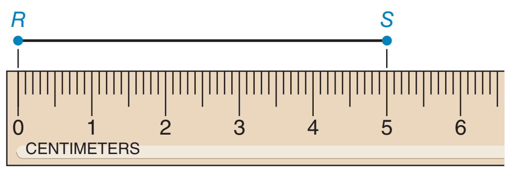


Figure P.24

When "0" is not placed at the endpoint of the line segment, the length of the line segment is the positive difference between the numbers found at its endpoints.

In Example 2, a ruler (not drawn to scale) is shown in Figure P.25. In the drawing, the distance between consecutive marks on the ruler corresponds to 1 inch. The measure of a line segment is known as *linear measure*.

EXAMPLE 2

In rectangle *ABCD* of Figure P.25, the line segments $\overline{AC}$ and $\overline{BD}$ shown are the diagonals of the rectangle. How do the lengths of the diagonals compare?

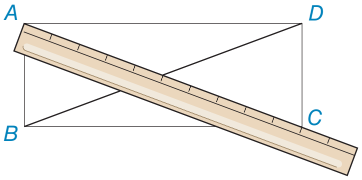


Figure P.25

SOLUTION As shown on the ruler, $AC = 10''$. As intuition suggests, the lengths of the diagonals are the same, so it follows that $BD = 10''$.

NOTE: In linear measure, 10″ means 10 inches, and 10′ means 10 feet.

Figure P.26

In Figure P.26, point *B* lies between *A* and *C* on $\overline{AC}$. If $AB = BC$, then *B* is the **midpoint** of $\overline{AC}$. When $AB = BC$, the geometric figures $\overline{AB}$ and $\overline{BC}$ are said to be **congruent;** in effect, geometric figures are congruent when one can be placed over the other (a perfect match). Numerical lengths may be equal, but the actual line segments (geometric figures) are congruent. The symbol for congruence is ≅; thus, $\overline{AB} \cong \overline{BC}$ if *B* is the midpoint of $\overline{AC}$. Example 3 emphasizes the relationship between $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$ when *B* lies between *A* and *C*.

SSG **EXS. 1–8**

EXAMPLE 3

In Figure P.27, the lengths of $\overline{AB}$ and $\overline{BC}$ are $AB = 4$ and $BC = 8$. What is *AC*, the length of $\overline{AC}$?

Figure P.27

SOLUTION As intuition suggests, the length of $\overline{AC}$ equals $AB + BC$. Thus, $AC = 4 + 8 = 12$.

Discover

The word *geometry* means the measure (from *metry*) of the earth (from *geo*). Words that contain *meter* also suggest the measure of some quantity. What is measured by each of the following objects?

odometer, pedometer, thermometer, altimeter, clinometer, anemometer

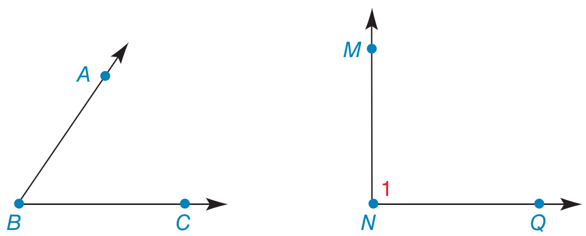
ANSWER

Vehicle mileage, distance walked, temperature, altitude, angle of inclination, wind speed

MEASURING ANGLES

Although we formally define an angle in Section 1.2, we consider it intuitively at this time.

An angle's measure depends not on the lengths of its sides but on the amount of opening between its sides. In Figure P.28, the arrows on the angles' sides suggest that the sides extend indefinitely.



(a) (b)

Figure P.28

The instrument shown in Figure P.29 (and used in the measurement of angles) is a **protractor**. For example, we would express the measure of $\angle RST$ by writing $m\angle RST = 50°$; this statement is read, "The measure of $\angle RST$ is 50 degrees." Measuring

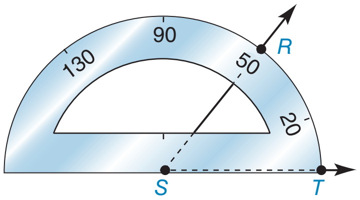


Figure P.29

the angles in Figure P.28 with a protractor, we find that m∠$B = 55°$ and m∠$1 = 90°$. If the degree symbol is missing, the measure is understood to be in degrees; thus, m∠$1 = 90$.

In practice, the protractor shown in Figures P.29 and P.30 will measure an angle that is greater than 0° but less than or equal to 180°.

To find the degree measure of an angle using a protractor:

1. Place the notch of the protractor at the point where the sides of the angle meet (the vertex of the angle). See point S in Figure P.30.
2. Place the protractor along a side of the angle so that the scale reads "0." See point *T* in Figure P.30 where we find "0" on the outer scale.
3. Using the same (outer) scale, read the angle size by reading the degree measure that corresponds to the second side of the angle.

Warning

Many protractors have dual scales, as shown in Figure P.30. To measure the angle, one must calculate the positive difference between two numbers on the same (inner or outer) scale.

EXAMPLE 4

For Figure P.30, find the measure of ∠*RST*.

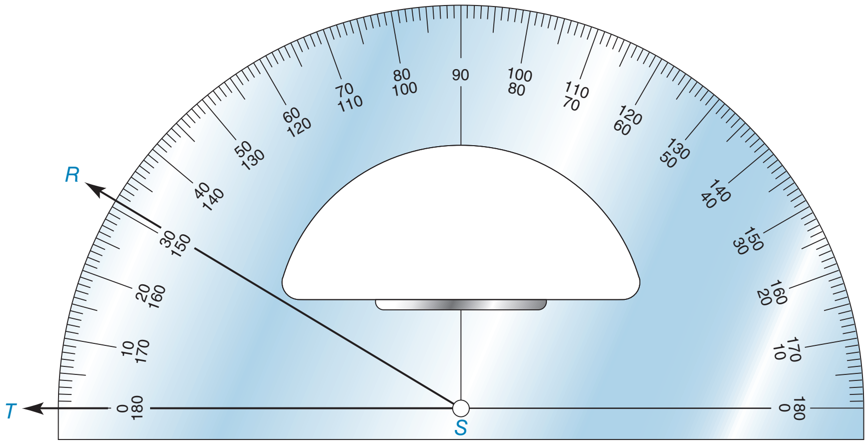


Figure P.30

SOLUTION Using the protractor, we find that the measure of angle *RST* is 31°. (In symbols, m∠*RST* = 31° or m∠*RST* = 31.)

Some protractors show a full 360°; such a protractor is used to measure an angle whose measure is between 0° and 360°. An angle whose measure is between 180° and 360° is known as a *reflex angle*.

Just as measurement with a ruler is not perfect, neither is measurement with a protractor.

The lines on a sheet of paper in a notebook are *parallel*. Informally, **parallel** lines lie on the same page and will not cross over each other even if they are extended indefinitely. We say that lines ℓ and m in Figure P.31(a) are parallel; note here the use of a lowercase letter to name a line. We say that line segments are parallel if they are parts of parallel lines; if $\overleftrightarrow{RS}$ is parallel to $\overleftrightarrow{MN}$, then $\overline{RS}$ is parallel to $\overline{MN}$ in Figure P.31(b).

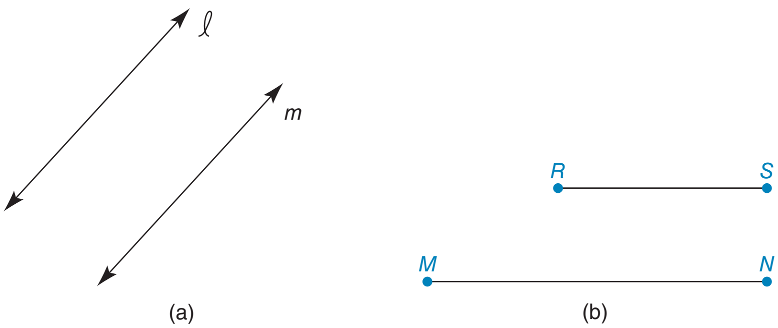


Figure P.31

The relationship between the parallel lines in Figure P.31(a) can be characterized by the statement $\ell \cap m = \varnothing$.

EXAMPLE 5

In Figure P.32, the sides of angles ABC and DEF are parallel ($\overline{AB}$ to $\overline{DE}$ and $\overline{BC}$ to $\overline{EF}$). Use a protractor to decide whether these angles have equal measures.

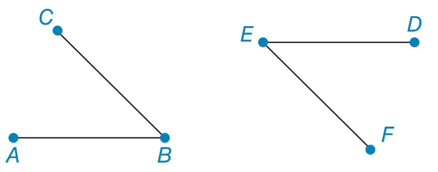


Figure P.32

SOLUTION The angles have equal measures. Both measure 44°.

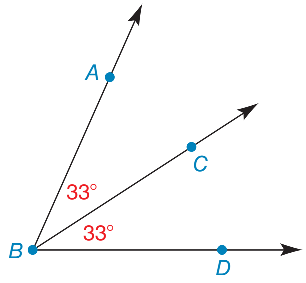


Figure P.33

Two angles with equal measures are said to be *congruent*. In Figure P.32, we see that $\angle ABC \cong \angle DEF$. In Figure P.33, $\angle ABC \cong \angle CBD$.

In Figure P.33, angle ABD has been separated into smaller angles ABC and CBD; if the two smaller angles are congruent (have equal measures), then angle ABD has been *bisected*. In general, the word **bisect** means to separate a line segment (or an angle) into two parts of equal measure; similarly, the word **trisect** means that the line segment (or angle) is separated into three parts of equal measure.

Any angle having a 180° measure is called a **straight angle**, an angle whose sides are in opposite directions. See straight angle RST in Figure P.34(a). When a straight angle is bisected, as shown in Figure P.34(b), the two angles formed are **right angles** (each measures 90°).

When two lines have a point in common, as in Figure P.35, they are said to **intersect**. When two lines intersect and form congruent adjacent angles, they are said to be **perpendicular**. In Figure P.35, lines r and t are perpendicular if $\angle 1 \cong \angle 2$ or $\angle 2 \cong \angle 3$, and so on.

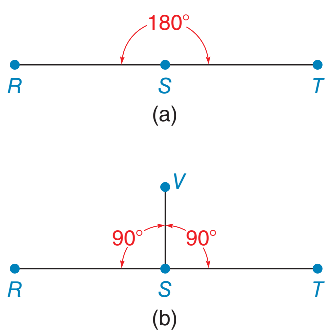


Figure P.34

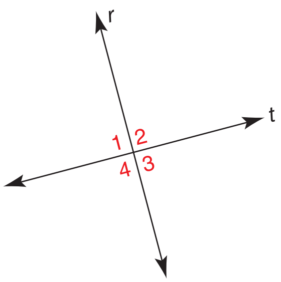


Figure P.35

EXAMPLE 6

In Figure P.35, suppose that **a)** $\angle 3 \cong \angle 4$ **b)** $\angle 1 \cong \angle 4$ **c)** $\angle 1 \cong \angle 3$. Are lines r and t perpendicular?

SSG **EXS. 9–13**

SOLUTION **a)** Yes **b)** Yes **c)** The lines could be perpendicular, but not necessarily.

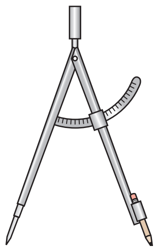


Figure P.36

CONSTRUCTIONS

Another tool used in geometry is the **compass**. This instrument, shown in Figure P.36, is used to draw circles and parts of circles known as *arcs*.

The ancient Greeks insisted that only two tools (a compass and a straightedge) be used for geometric **constructions**, which were idealized drawings assuming perfection in the use of these tools. The compass was used to create "perfect" circles and for marking off segments of "equal" length. The straightedge could be used to draw a straight line through two designated points.

A **circle** is the set of all points in a plane that are at a given distance from a particular point (known as the "center" of the circle). The part of a circle between any two of its points is known as an **arc**. Any line segment joining the center to a point on the circle is a **radius** (plural: *radii*) of the circle. See Figure P.37.

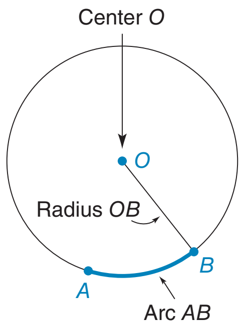


Figure P.37

Construction 1, which follows, is quite basic and depends only on using arcs of the same radius length to construct line segments of the same length. The arcs are created by using a compass. Construction 2 is more difficult to perform and explain, so we will delay its explanation to a later chapter (see Section 3.4).

CONSTRUCTION 1 Construct a segment congruent to a given segment.

GIVEN: $\overline{AB}$ in Figure P.38(a)

CONSTRUCT: $\overline{CD}$ on line m so that $\overline{CD} \cong \overline{AB}$ (or $CD = AB$)

CONSTRUCTION: With your compass open to the length of $\overline{AB}$, place the stationary point of the compass at C and mark off a length equal to AB at point D, as shown in Figure P.38(b). Then $CD = AB$.

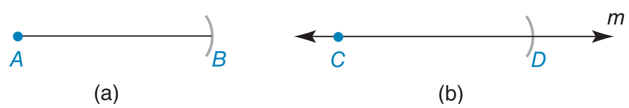


Figure P.38

The following construction is shown step by step in Figure P.39. Intuition suggests that point M in Figure P.39(c) is the midpoint of $\overline{AB}$.

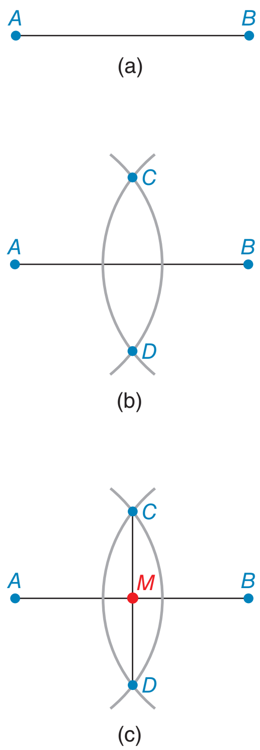


Figure P.39

CONSTRUCTION 2 Construct the midpoint M of a given line segment AB.

GIVEN: $\overline{AB}$ in Figure P.39(a)

CONSTRUCT: M on $\overline{AB}$ so that $AM = MB$

CONSTRUCTION: Figure P.39(a): Open your compass to a length greater than one-half of $\overline{AB}$.

Figure P.39(b): Using A as the center of the arc, mark off an arc that extends both above and below segment AB. With B as the center and keeping the same length of radius, mark off an arc that extends above and below $\overline{AB}$ so that two points (C and D) are determined where the arcs cross.

Figure P.39(c): Now draw $\overline{CD}$. The point where $\overline{CD}$ crosses $\overline{AB}$ is the midpoint M.

SSG EXS. 14–17

EXAMPLE 7

In Figure P.40, M is the midpoint of $\overline{AB}$.

a) Find AM if $AB = 15$.
b) Find AB if $AM = 4.3$.
c) Find AB if $AM = 2x + 1$.

Figure P.40

SOLUTION

a) AM is one-half of AB, so $AM = 7\frac{1}{2}$.
b) AB is twice AM, so $AB = 2(4.3)$ or $AB = 8.6$.
c) AB is twice AM, so $AB = 2(2x + 1)$ or $AB = 4x + 2$.

The technique from algebra used in Example 8 and also needed for Exercises 47 and 48 of this section depends on the following properties of addition and subtraction.

If $a = b$ and $c = d$, then $a + c = b + d$.
Words: Equals added to equals provide equal sums.
Illustration: Since $0.5 = \frac{5}{10}$ and $0.2 = \frac{2}{10}$, it follows that $0.5 + 0.2 = \frac{5}{10} + \frac{2}{10}$; that is, $0.7 = \frac{7}{10}$.

If $a = b$ and $c = d$, then $a - c = b - d$.
Words: Equals subtracted from equals provide equal differences.
Illustration: Since $0.5 = \frac{5}{10}$ and $0.2 = \frac{2}{10}$, it follows that $0.5 - 0.2 = \frac{5}{10} - \frac{2}{10}$; that is, $0.3 = \frac{3}{10}$.

EXAMPLE 8

Figure P.41

In Figure P.41, point B lies on $\overline{AC}$ between A and C. If $AC = 10$ and AB is 2 units longer than BC, find the length x of $\overline{AB}$ and the length y of $\overline{BC}$.

SOLUTION

Because $AB + BC = AC$, we have $x + y = 10$.
Because $AB - BC = 2$, we have $x - y = 2$.
Adding the left and right sides of these equations, we have

$$\begin{array}{r} x + y = 10 \\ x - y = 2 \\ \hline 2x \quad = 12 \end{array} \qquad \text{so } x = 6.$$

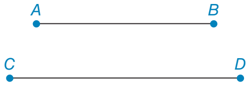
If $x = 6$, then $x + y = 10$ becomes $6 + y = 10$ and $y = 4$.
Thus, $AB = 6$ and $BC = 4$.

SSG **EXS. 18, 19**

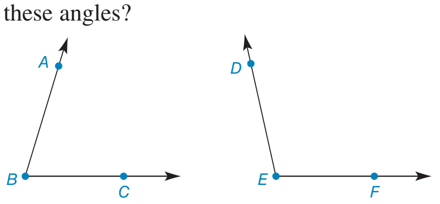
Exercises P.3

Note: Exercises preceded by an asterisk are of a more challenging nature.

1. If line segment AB and line segment CD are drawn to scale, what does intuition tell you about the lengths of these segments?



2. If angles ABC and DEF were measured with a protractor, what does intuition tell you about the degree measures of these angles?



3. How many endpoints does a line segment have? How many midpoints does a line segment have?

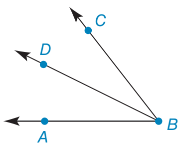
4. Do the points *A*, *B*, and *C* appear to be collinear?

Exercises 4–6

5. How many lines can be drawn that contain both points *A* and *B*? How many lines can be drawn that contain points *A*, *B*, and *C*?

6. Consider noncollinear points *A*, *B*, and *C*. If each line must contain two of the points, what is the total number of lines that are determined by these points?

7. Name all the angles in the figure.

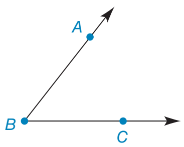


8. Which of the following measures can an angle have? 23°, 90°, 200°, 110.5°, −15°

9. Must two different points be collinear? Must three or more points be collinear? Can three or more points be collinear?

10. Which symbol(s) correctly expresses the order in which the points *A*, *B*, and *X* lie on the given line, *A-X-B* or *A-B-X*?

11. Which symbols correctly name the angle shown? ∠*ABC*, ∠*ACB*, ∠*CBA*



12. A triangle is named Δ*ABC*. Can it also be named Δ*ACB*? Can it be named Δ*BAC*?

13. Consider rectangle *MNPQ*. Can it also be named rectangle *PQMN*? Can it be named rectangle *MNQP*?

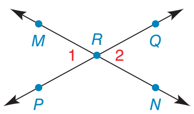
14. Suppose ∠*ABC* and ∠*DEF* have the same measure. Which statements are expressed correctly?

a) m∠*ABC* = m∠*DEF* b) ∠*ABC* = ∠*DEF*
c) m∠*ABC* ≅ m∠*DEF* d) ∠*ABC* ≅ ∠*DEF*

15. Suppose $\overline{AB}$ and $\overline{CD}$ have the same length. Which statements are expressed correctly?

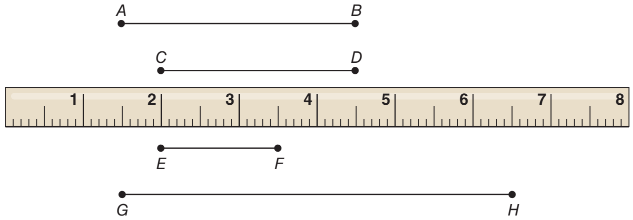
a) *AB* = *CD* b) $\overline{AB} = \overline{CD}$
c) *AB* ≅ *CD* d) $\overline{AB} \cong \overline{CD}$

16. When two lines cross (intersect), they have exactly one point in common. In the drawing, what is the point of intersection? How do the measures of ∠1 and ∠2 compare?



17. Judging from the ruler shown (not to scale), estimate the measure of each line segment.

a) *AB* b) *CD*



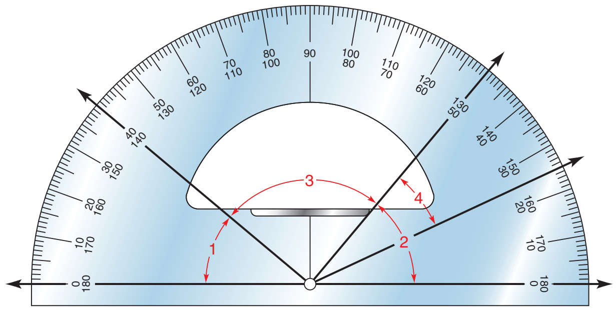
Exercises 17, 18

18. Judging from the ruler, estimate the measure of each line segment.

a) *EF* b) *GH*

19. Judging from the protractor provided, estimate the measure of each angle to the nearest multiple of 5° (such as, 20°, 25°, 30°, etc.).

a) m∠1 b) m∠3

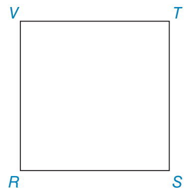


Exercises 19, 20

20. Using the drawing for Exercise 19, estimate the measure of each angle to the nearest multiple of 5° (such as, 20°, 25°, 30°, etc.).

a) m∠2 b) m∠4

21. Consider the square at the right, *RSTV*. It has four right angles and four sides of the same length. How are sides $\overline{RS}$ and $\overline{ST}$ related? How are sides $\overline{RS}$ and $\overline{VT}$ related?



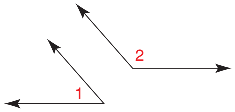
Exercises 21, 22

22. Square *RSTV* has diagonals $\overline{RT}$ and $\overline{SV}$ (not shown). If the diagonals are drawn, how will their lengths compare? Do the diagonals of a square appear to be perpendicular?

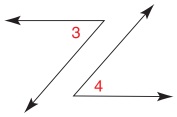
23. Use a compass to draw a circle. Draw a radius, a line segment that connects the center to a point on the circle. Measure the length of the radius. Draw other radii and find their lengths. How do the lengths of the radii compare?

24. Use a compass to draw a circle of radius 1 inch. Draw a chord, a line segment that joins two points on the circle. Draw other chords and measure their lengths. What is the largest possible length of a chord in this circle?

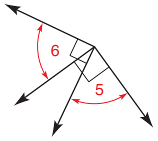
25. The sides of the pair of angles are parallel. Are ∠1 and ∠2 congruent?



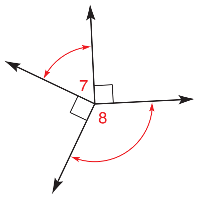
26. The sides of the pair of angles are parallel. Are ∠3 and ∠4 congruent?



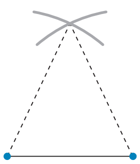
27. The sides of the pair of angles are perpendicular. Are ∠5 and ∠6 congruent?



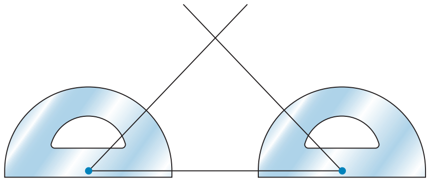
28. The sides of the pair of angles are perpendicular. Are ∠7 and ∠8 congruent?



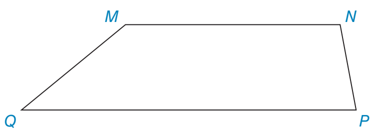
29. On a piece of paper, use your compass to construct a triangle that has two sides of the same length. Cut the triangle out of the paper and fold the triangle in half so that the congruent sides coincide (one lies over the other). What seems to be true of two angles of that triangle?



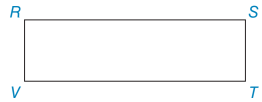
30. On a piece of paper, use your protractor to draw a triangle that has two angles of the same measure. Cut the triangle out of the paper and fold the triangle in half so that the angles of equal measure coincide (one lies over the other). What seems to be true of two of the sides of that triangle?



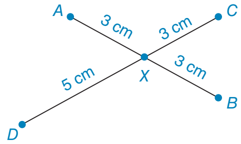
31. A trapezoid is a four-sided figure that contains one pair of parallel sides. Which sides of the trapezoid *MNPQ* appear to be parallel?



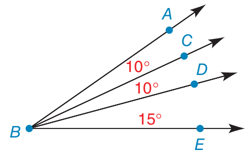
32. In the rectangle shown, what is true of the lengths of each pair of opposite sides?



33. A line segment is bisected if its two parts have the same length. Which line segment, $\overline{AB}$ or $\overline{CD}$, is bisected at point *X*?



34. An angle is bisected if its two parts have the same measure. Use three letters to name the angle that is bisected.



In Exercises 35 to 38, with A-B-C on $\overline{AC}$, it follows that $AB + BC = AC$.

Exercises 35–38

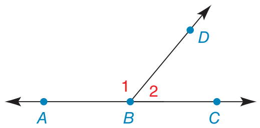
35. Find AC if $AB = 9$ and $BC = 13$.

36. Find AB if $AC = 25$ and $BC = 11$.

37. Find x if $AB = x$, $BC = x + 3$, and $AC = 21$.

38. Find an expression for AC (the length of $\overline{AC}$) if $AB = x$ and $BC = y$.

39. ∠*ABC* is a straight angle. Using your protractor, you can show that $m\angle 1 + m\angle 2 = 180°$. Find $m\angle 1$ if $m\angle 2 = 56°$.



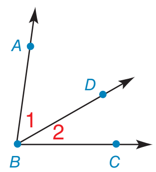
Exercises 39, 40

40. Find $m\angle 1$ if $m\angle 1 = 2x$ and $m\angle 2 = x$.

(HINT: See Exercise 39.)

In Exercises 41 to 44, $m\angle 1 + m\angle 2 = m\angle ABC$.

41. Find $m\angle ABC$ if $m\angle 1 = 42°$ and $m\angle 2 = 29°$.



Exercises 41–44

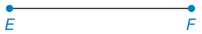
42. Find $m\angle 1$ if $m\angle ABC = 68°$ and $m\angle 1 = m\angle 2$.

43. Find x if $m\angle 2 = x$, $m\angle 1 = 2x + 3$, and $m\angle ABC = 72°$.

44. Find an expression for $m\angle ABC$ if $m\angle 1 = x$ and $m\angle 2 = y$.

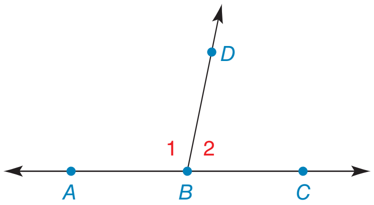
45. A compass was used to mark off three congruent segments, $\overline{AB}$, $\overline{BC}$, and $\overline{CD}$. Thus, $\overline{AD}$ has been trisected at points B and C. If $AD = 32.7$, how long is $\overline{AB}$?

46. Use your compass and straightedge to bisect $\overline{EF}$.



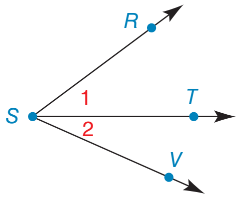
***47.** In the figure, $m\angle 1 = x$ and $m\angle 2 = y$. If $x - y = 24°$, find x and y.

(*HINT:* $m\angle 1 + m\angle 2 = 180°$.)



***48.** In the drawing, $m\angle 1 = x$ and $m\angle 2 = y$. If $m\angle RSV = 67°$ and $x - y = 17°$, find x and y.

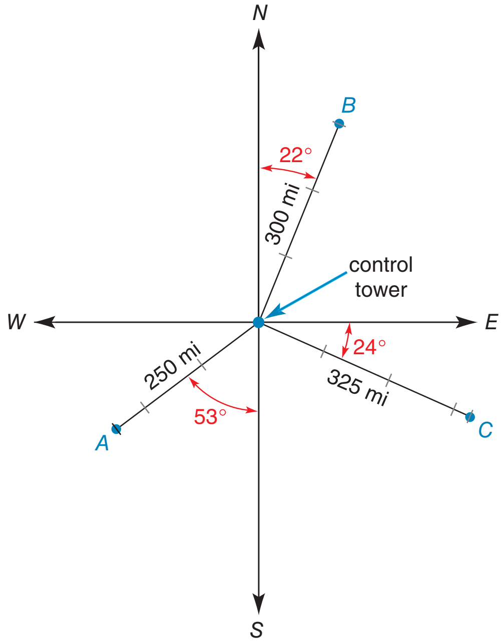
(*HINT:* $m\angle 1 + m\angle 2 = m\angle RSV$.)



For Exercises 49 and 50, use the following information. Relative to its point of departure or some other point of reference, the angle that is used to locate the position of a ship or airplane is called its bearing. The bearing may also be used to describe the direction in which the airplane or ship is moving. By using an angle between 0° and 90°, a bearing is measured from the North-South line toward the East or West. In the diagram, airplane A (which is 250 miles from Chicago's O'Hare airport's control tower) has a bearing of S 53° W.

49. Find the bearing of airplane B relative to the control tower.

50. Find the bearing of airplane C relative to the control tower.



Exercises 49, 50

PERSPECTIVE ON HISTORY

OUR GREEK HERITAGE

In the word geometry, the prefix "geo" refers to the earth, while the suffix "metry" refers to measure. Thus, our study of geometry enables us to measure on the earth and from the earth as well. Much of the geometry knowledge and technique that we study today can be traced to the ancient Greeks. Thales (640 B.C.–550 B.C.), known as one of the seven sages of Greece, founded the earliest school of mathematics. Some of the earliest collections of mathematical knowledge were written by Eudemus (around 325 B.C., a pupil of Aristotle), Geminus (around 50 B.C., comparing methods of proof used by Greek geometers), and Proclus (around 450 A.D., commenting upon Euclid's *Elements*). Euclid (330 B.C.–275 B.C.) was credited with the title "Father of Geometry" in that he formalized its study in his *Elements*, a textbook for the student of geometry. Due to the vast content of geometry knowledge and the challenges found in its relationships, Euclid, a teacher with compassion, is credited with the statement "There is no royal road to geometry." In contrast to Euclid, Archimedes (287 B.C.–212 B.C.) was practical, a mechanical engineer, and an inventor of numerous mechanisms whose designs depended upon a vast knowledge of geometry. Among Archimedes's discoveries were a method for the detection of gold (or lack thereof), the use of mirrors to create heat, the use of cogs and screws to move large masses, and the catapult as a weapon. Although the development of geometry continues to this day, many of the ideas that you find in this textbook can be attributed to the ancient Greeks mentioned herein.

PERSPECTIVE ON APPLICATIONS

ONE-TO-ONE CORRESPONDENCE

As small children, we were taught to count. . . 1, 2, 3, and so on. With time, we began to count objects by associating these counting numbers in order and one count for each distinct object in a set—a concept known as *one-to-one correspondence*. Children count the number of My Little Ponies or Hot Wheels or Blu-Ray discs or video games. Looking out the patio door while holding his grandson, a grandfather asks the grandson, "How many birds are on the bird feeder?"; Old enough to utilize the notion of one-to-one correspondence, the grandson correctly says that there are three birds on the feeder. See Figure P.42.

Next, we consider the uncommon question, "Can birds also count people?" Here's a story, showing that at least some of them can:

> *A farmer in England found that every day when he went into his blind in the field, the crows would stay away. But once he left, they would fly into the fields and feed. He brought a friend with him one day to follow him into the blind, thinking the crows would spot one of them leaving and then come down to eat his crops, thinking that no one was watching.*
>
> *But the crows knew that a second man was still in the blind. The next day they tried it with another man, then another. The crows counted and subtracted. They knew that someone was still waiting for them in the blind. It was only after they reached 16 men that the crows lost count.*

SOURCE: Nancy Bennett, "Crows Can Count—at Least to 16." *Capper's Farmer*, January 2006. https://www.cappersfarmer.com/humor-and-nostalgia/crows-can-count---at-least-to-16

More scientifically, researchers at the University of Tübingen found that crows are very smart birds. They can use tools much like humans to solve some problems that are too tough for five-year-old children.


Ancha Chiangmai/Shutterstock.com

Figure P.42

Summary

A Look Back at Chapter P

This preliminary chapter was included for those students who have not studied sets, set relationships, statements and types of reasoning, and finally some basic terms from geometry. Of course, geometric figures are sets of points. In later chapters, the principles of geometry will be written in the form of statements. Types of reasoning, particularly deduction, will be extremely important in developing and verifying those geometric properties that will be called theorems. Basic terms such as line, angle, and triangle were included in the final section; these, and many other terms, will be given a more formal treatment in Chapter 1. In Chapter P, we also considered the use of tools such as the ruler, protractor, and compass.

A Look Ahead to Chapter 1

Our chapter begins with the fact that geometry is a mathematical system. The four components of any mathematical system include undefined terms, defined terms (definitions), postulates (statements that are accepted as true), and theorems (statements that we can verify are true). Undefined terms provide the building blocks necessary to define other terms. Postulates also provide a basis for those theorems that we will "prove" in this and later chapters.

Key Concepts

P.1

Set • Element • Finite and Infinite Sets • Subset • Point Paths (Straight, Curved, Circular, Scattered) • Between • Continuous/Discontinuous • Line Segment, Line, Ray • Intersection and Union of Sets • Empty Set • Angle • Disjoint Sets • Venn Diagrams • Equivalent Sets • Universe • Complement of Set

P.2

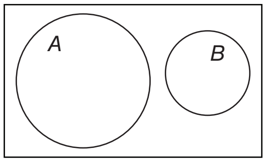
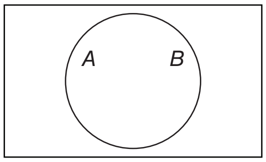
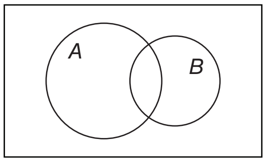
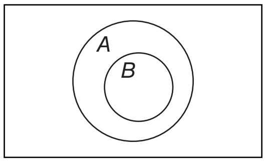
Statement • Variable • Negation • Compound Statement • Conjunction • Disjunction • Conditional Statement (Implication) • Hypothesis • Conclusion • Reasoning • Intuition • Induction • Deduction • Valid and Invalid Arguments • Law of Detachment • Counterexample

P.3

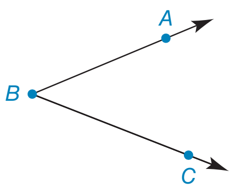
Point • Line • Collinear Points • Vertex • Line Segment • Midpoint • Congruent • Protractor • Parallel • Bisect • Trisect • Straight Angle • Right Angle • Intersect • Perpendicular • Compass • Constructions • Circle • Arc • Radius

Overview ◆ Chapter P

Set Relationships

Figure	Relationship	Symbol
	*A* and *B* are disjoint	$A \cap B = \varnothing$
	*A* and *B* are equivalent	$A = B$
	*A* and *B* intersect	$A \cap B \neq \varnothing$
	*B* is a subset of *A*	$B \subseteq A$

Geometric Figures and Symbols

Figure	Description	Symbol
	Line *AB*	$\overleftrightarrow{AB}$
	Line segment *AB*	$\overline{AB}$
	Ray *AB*	$\overrightarrow{AB}$
	Angle *ABC* (or *CBA*)	$\angle ABC$ (or $\angle CBA$)

(continued)

Types of Reasoning

Type	*Characterization*	*Example*
Intuition	A quickly drawn conclusion; insight	(C) We will lose the game tomorrow.
Induction	A conclusion based upon numerous observations.	(C) Because Mary was late to school the past three days, she will be late for school tomorrow.
Deduction	A conclusion that follows logically from given information.	(1) If I study, then I will pass the test. (2) I study for the test. (C) I pass the test.

Chapter P Review Exercises

For Review Exercises 1 to 6, which relationship (disjoint, equivalent, subset, intersect) exists between the two sets described?

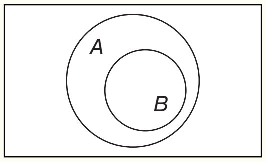
1. $A = \{1,2,3,4,5\}$; $B = \{2,4,6,8,10\}$

2. $A = \{1,2,3,4,5\}$; $B = \{6,7,8,9,10\}$

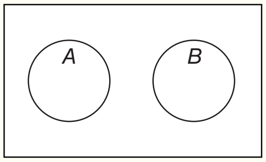
3. $A = \{\text{vowels}\}$; $B = \{\text{consonants}\}$

4. $A = \{\text{vowels}\}$; $B = \{a,e,i,o,u\}$

5.



6.



7. Give another name for $\overline{AB}$ (not shown).

8. Give another name for $\angle ABC$ (not shown).

9. Find: $N\{\text{positive odd integers less than } 20\}$

10. Find: $N\{\text{sides of a triangle}\} + N\{\text{sides of a quadrilateral}\}$

11. If $N\{A\} = 37$, $N\{B\} = 43$, and $N\{A \cap B\} = 15$, find $N\{A \cup B\}$.

12. If $N\{A \cup B\} = 69$, $N\{A\} = 35$, and $N\{B\} = 47$, find $N\{A \cap B\}$.

For Review Exercises 13 to 18, name the type of reasoning (intuition, induction, deduction) used.

13. While waiting to bat in a baseball game, Phillip thinks, "I'll be able to hit against that pitcher."

14. Emma knows that she will have to wear a jacket to school if it is cool. Her mother tells her that the weather is freezing. Emma knows that she will need to wear her jacket to school.

15. Laura is at camp. On the first day, her mother brings her some clothing. On the second day, her mother brings her another pair of shoes. On the third day, her mother brings her cookies. Laura concludes that her mother will bring her something on the fourth day of camp.

16. Without tasting her food, Zaidah says "I love this stuff."

17. Sarah knows the rule "Any number (not 0) divided by itself equals one." The teacher asks Sarah, "What is 5 divided by 5?" Sarah says, "The answer is 1."

18. Alice goes to the grocery store to shop for supper. In the meat department, she sees that ribeye steak sells for $7.99 per pound. Another cut of steak sells for $9.50 per pound. Lobster is priced at $11.99 per pound. Alice concludes that the prices for meat and seafood are just too high.

For Review Exercises 19 and 20, statements P and Q are true while statement R is false. Classify each statement as true or false.

19. a) P and R b) $\sim R$ or Q

20. a) $\sim Q$ or R b) P and $\sim R$

For Review Exercises 21 to 26, assume that statements 1 and 2 are true. Draw a conclusion where possible.

21. 1) If a person has a good job, then that person has a college degree.
2) Henry has a college degree.
C) ?

22. 1) If a person has a good job, then that person has a college degree.
2) Meg has a good job.
C) ?

23. 1) If an angle has a measure of 90 degrees, then the angle is a right angle.
2) Angle ABC has a measure of 90 degrees.
C) ?

24. 1) If an angle is a right angle, then it has a measure of 90 degrees.
2) Angle DEF is a right angle.
C) ?

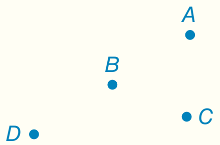
25. 1) If Mara goes to an Italian restaurant for dinner, she will order the chicken broccoli alfredo.
2) Mara goes to Red Lobster seafood restaurant for dinner.
C) ?

26. 1) If Don wants a straight cut, he uses his power miter saw.
2) Don wants to get a straight cut on a piece of trim board.
C) ?

For Review Exercises 27 and 28, use the drawing below. Classify statements as true or false.

27. a) Points A, B, and C are collinear.
b) Points A, B, and D are collinear.

28. a) If A, B, and D are collinear, then B is between A and D.
b) B is between A and C.

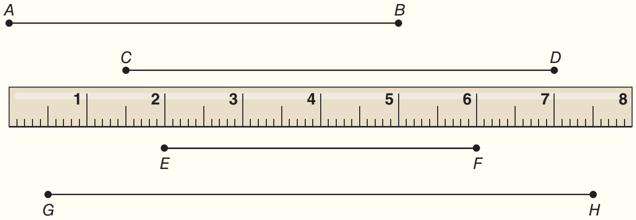


Exercises 27, 28

For Review Exercises 29 and 30, use the drawing below. Find each indicated length.

29. a) AB b) CD

30. a) EF b) GH

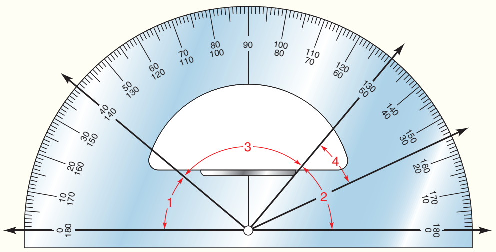


Exercises 29, 30

For Review Exercises 31 and 32, use the drawing below. Find the measure of each indicated angle.

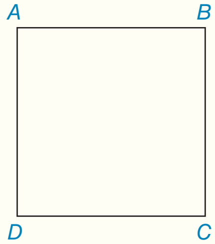
31. a) $\angle 1$ b) $\angle 3$

32. a) $\angle 2$ b) $\angle 4$

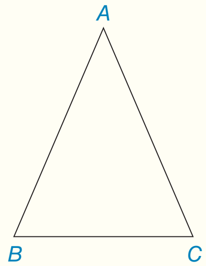


Exercises 31, 32

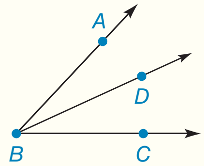
33. Given that $ABCD$ is a "square," use intuition to draw a conclusion regarding the lengths of $\overline{AC}$ and $\overline{BD}$.



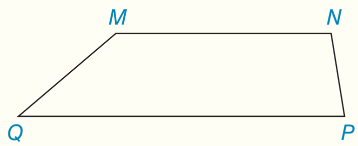
34. Given that $AB = AC$ in triangle ABC, use intuition to draw a conclusion regarding $m\angle ABC$ and $m\angle ACB$.



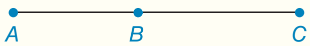
35. Given that $\overrightarrow{BD}$ "bisects" $\angle ABC$, use intuition to draw a conclusion regarding $m\angle ABD$ and $m\angle ABC$.



36. Given that $MNPQ$ is a "trapezoid," use intuition to draw a conclusion regarding $\overline{MN}$ and $\overline{PQ}$.



37. In the figure, A-B-C. If $AB = 6.2$ and $BC = 9.5$, find AC.



Exercises 37–39

38. In the figure, A-B-C. If $AC = 12.3$ and $BC = 7.4$, find AB.

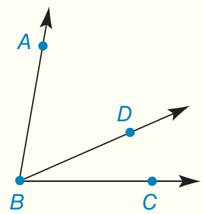
39. In the figure, A-B-C. If $AB = x$, $BC = x + 7$, and $AC = 4x - 3$, find x.

40. In the figure, $m\angle ABD = 31°$ and $m\angle DBC = 26°$. Find $m\angle ABC$.

41. In the figure, $m\angle ABC = 63°$ and $m\angle ABD = 39°$. Find $m\angle DBC$.

42. In the figure, $m\angle ABD = 2y + 7$, $m\angle DBC = y + 10$, and $m\angle ABC = 62°$. Find y.

43. In the figure, $m\angle ABD = x + 5$ and $m\angle DBC = 2x - 26$. If $\overrightarrow{BD}$ bisects $\angle ABC$, find x.



Exercises 40–43

Chapter P Test

For Exercises 1 and 2, let $A = \{1,2,3,4,5\}$, $B = \{2,4,6,8,10\}$, *and* $C = \{2,3,5,7,11\}$.

1. Find $(A \cup B) \cap C$.

2. Find $(A \cap B) \cup (A \cap C)$.

3. Give another name for:
a) $\overleftrightarrow{AB}$ b) $\angle ABC$

4. If $N\{A\} = 31$, $N\{B\} = 47$, and $N\{A \cap B\} = 17$, find $N\{A \cup B\}$.

5. At Rosemont High School, 14 players are on the varsity basketball team, 35 players are on the varsity football team, and 7 of these players are on both teams. How many different individual players are on the two varsity teams?

6. Name the type of reasoning used in the following scenario. While shopping for a new television, Henry finds that each of the first five TVs he sees are “smart” TVs. Before looking at the sixth TV, Henry concludes that the sixth set will also be a “smart” TV.

For Exercises 7 and 8, state a conclusion when possible.

7. 1) If a person studies geometry, then he/she will develop reasoning skills.
2) Kianna is studying geometry this semester.
C) ______________________________

8. 1) All major league baseball players enjoy a six-figure annual salary.
2) Nickolas makes a six-figure annual salary.
C) ____

9. Let A be any set of objects. Find expressions for:
a) $A \cup \varnothing$ (b) $A \cap \varnothing$

10. Statements P and Q are true while R is a false statement. Classify as true or false:
a) P or $\sim Q$ b) (P and Q) or R

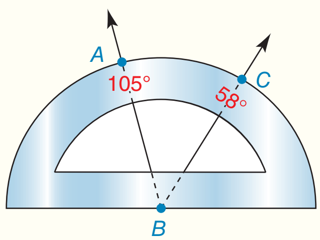
For Exercises 11 and 12, use the drawing provided.

Exercises 11, 12

11. If $AB = 11.8$ and $AX = 6.9$, find XB.

12. If $AX = x + 3$, $XB = x$, and $AB = 3x - 7$, find x.

13. Use the protractor with measures as indicated to find m$\angle ABC$.

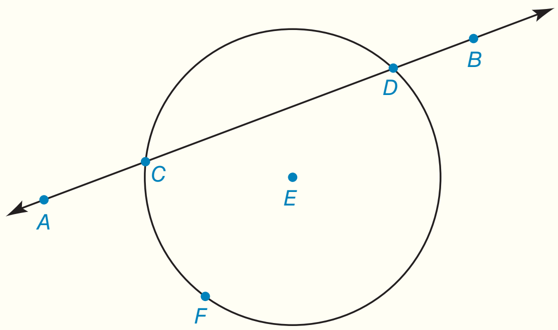


14. Classify each compound statement as true or false.
a) Rain is wet or snow is cold.
b) If Tom lives in Chicago, then Tom lives in Illinois.

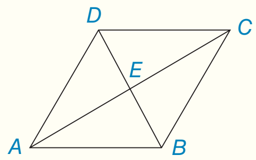
15. a) Which of these ($\overleftrightarrow{AB}$, $\overline{AB}$, or AB) represents the length of line segment AB?
b) Which (m$\angle CBA$, m$\angle CAB$, or m$\angle BAC$) represents the measure of $\angle ABC$?

16. Let P represent any statement. Classify as true or false.
a) P and $\sim P$ b) P or $\sim P$

17. In the figure shown, find the set of points that is the intersection of the circle and line AB.



18. Given “rhombus” $ABCD$, use intuition to draw a conclusion regarding diagonals $\overline{AC}$ and $\overline{DB}$.



19. For $\angle ABC$ (not shown), ray $\overrightarrow{BD}$ is the “bisector” of the angle. If m$\angle DBC = 27°$, find m$\angle ABC$.

20. In the figure shown, $\overline{CD}$ bisects $\overline{AB}$ at point M so that $AM = MB$. Is it correct to conclude that $CM = MD$?